CaP turing mAths

CONCRETE PICTORIAL ABSTRACT

2 + 1 = 3

Calculation Policy
Autumn 2016

This policy has been designed to teach children through the use of concrete, pictorial and abstract methods. This calculation policy should be used to support children to develop a deep understanding of number and calculation.

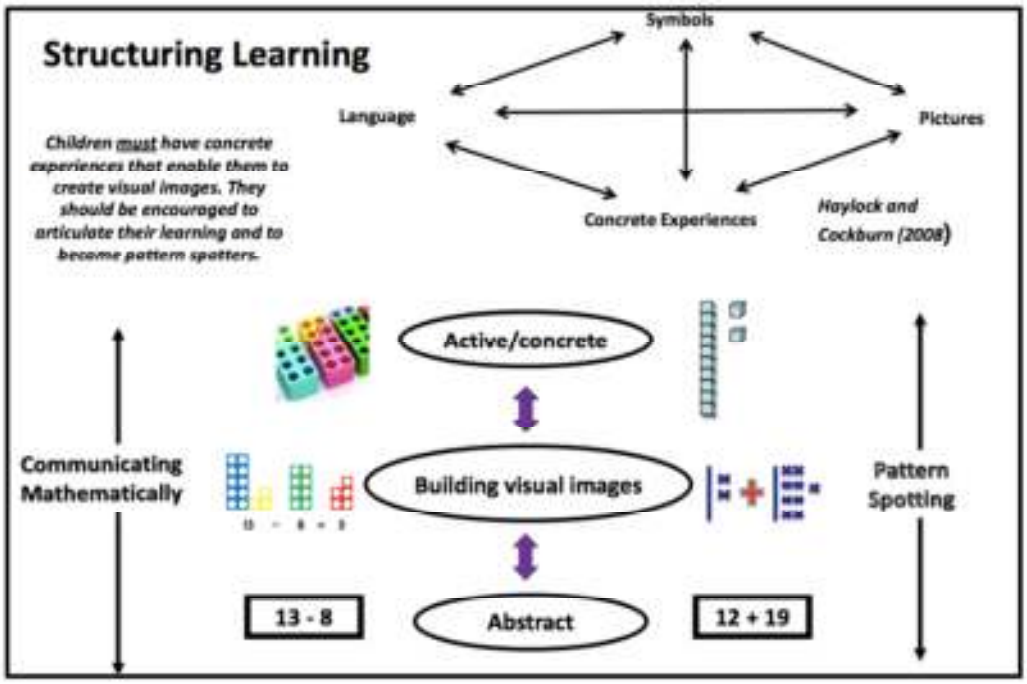
Background
This policy has been developed by Maths Coordinators with a specific interest in the use of Singapore methods to develop number awareness and fluency.
The policy only details the strategies; teachers must plan opportunities for pupils to apply these; for example, when solving problems, or where opportunities emerge elsewhere in the curriculum.

Using the concrete-pictorial-abstract approach:
Children develop an understanding of a mathematical concept through the three steps (or representation) of concrete-pictorial-abstract approach. Reinforcement is achieved by going back and forth between these representations.

Concrete representation The enactive stage - a pupil is first introduced to an idea or a skill by acting it out with real objects. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

Pictorial representation The iconic stage - a pupil has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

Abstract representation The symbolic stage - a pupil is now capable of representing problems by using mathematical notation, for example: 12 ÷ 2 = 6 .

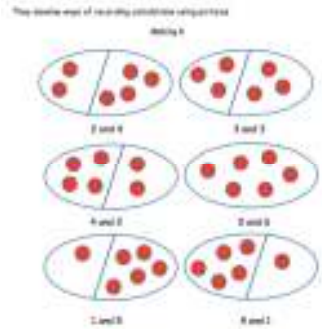


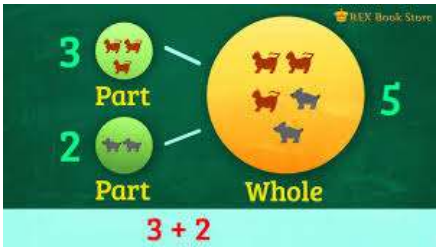
Guidance
This is document provides guidance and examples for key objectives for each year group but is not to be followed as a complete planning aid as not all objectives are exemplified.

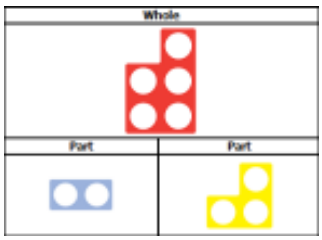
Reception

Addition

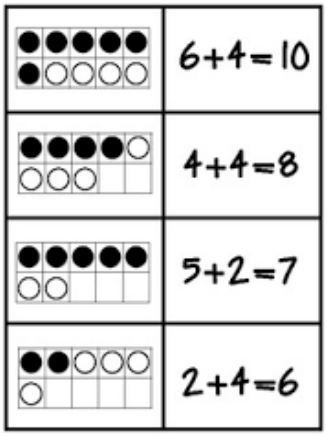
Explore part part whole relationship



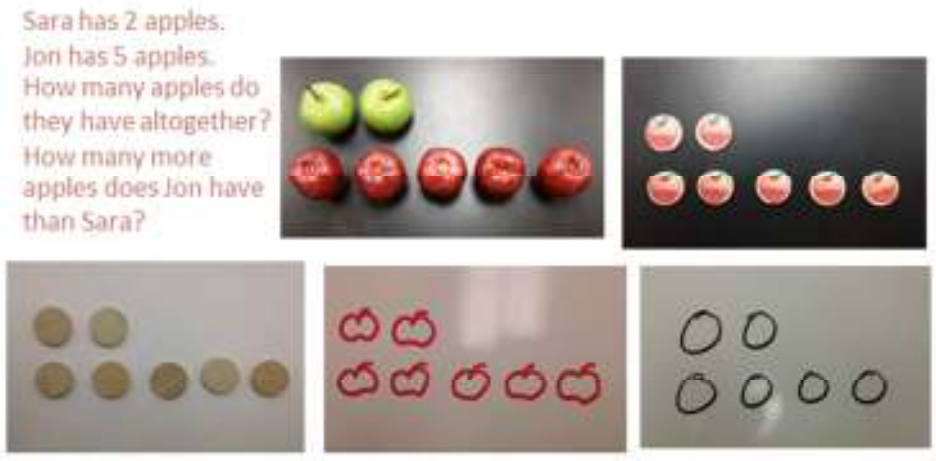




Using the ten frame to support addition of single digits – counting all/combining two groups



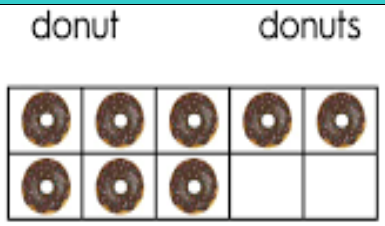
Solving problems using concrete and pictorial images.



Subtraction

Taking away after counting out practical equipment. . Children would be encouraged to physically remove these using touch counting.

By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.

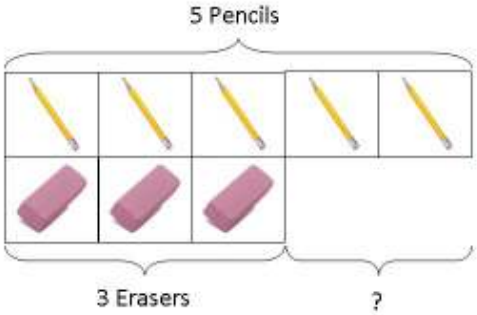


8 − 4 = __

Those who are ready may record their own calculations

0	1	2	3	4	5	6	7	8	9	10

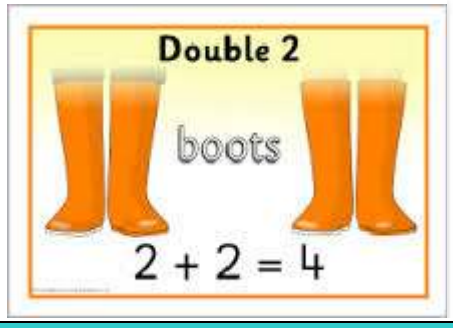
Using the ten frame to support subtraction by taking away



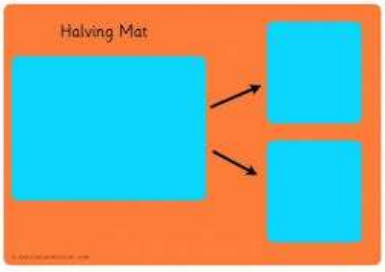
- Peter has 5 pencils and 3 erasers. How many more pencils than erasers does he have?

Solving problems using concrete and pictorial images.

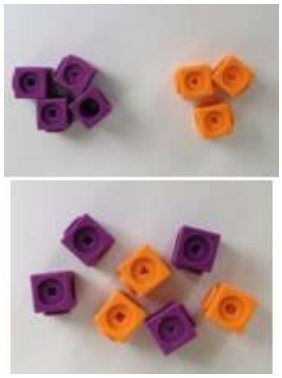
Multiplication

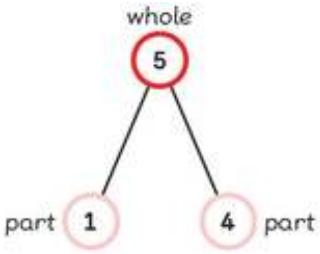
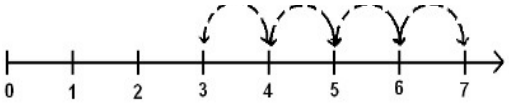
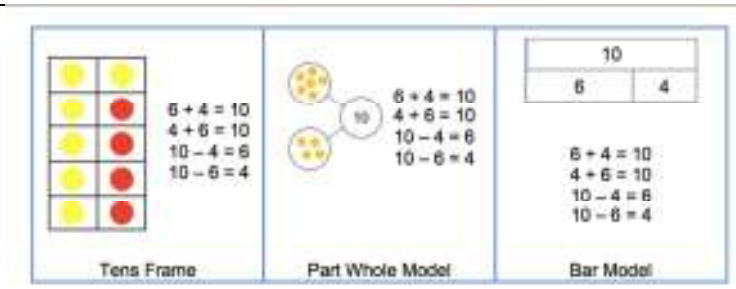
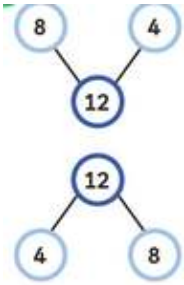
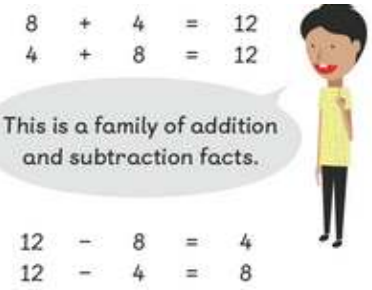


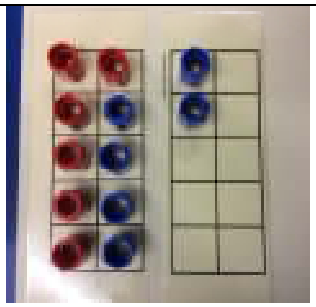
Division



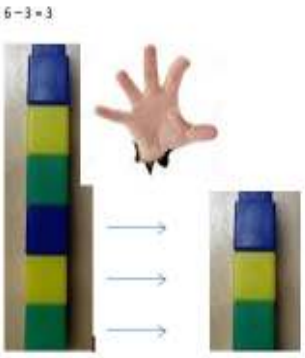
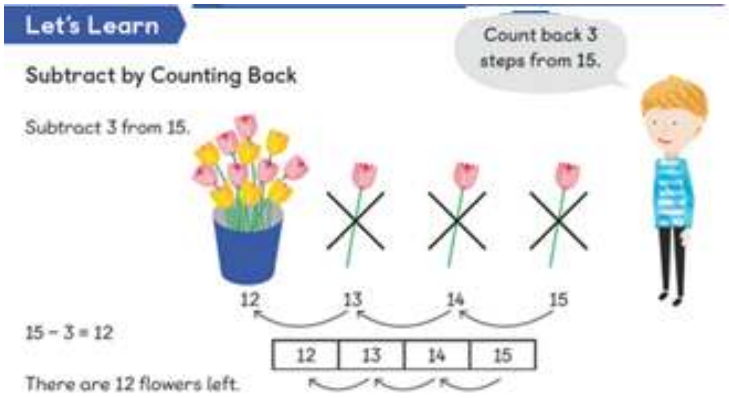
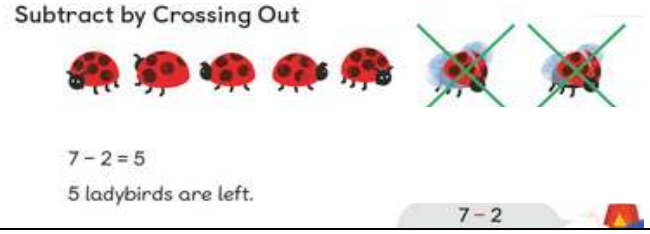
Year 1

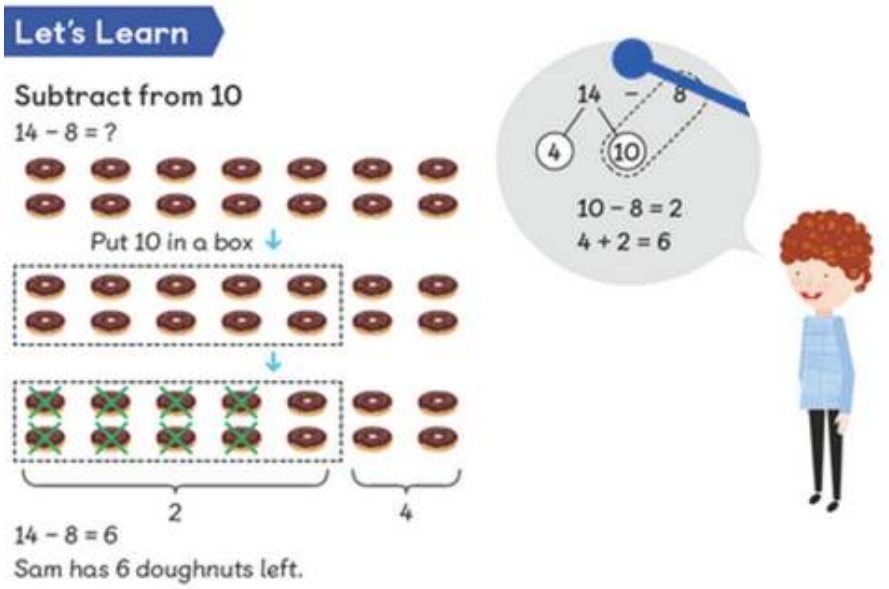
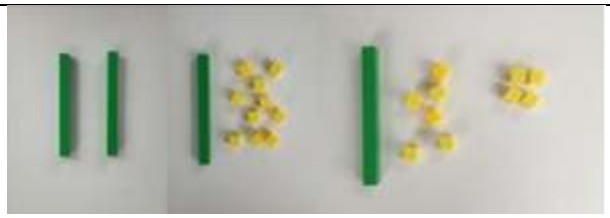
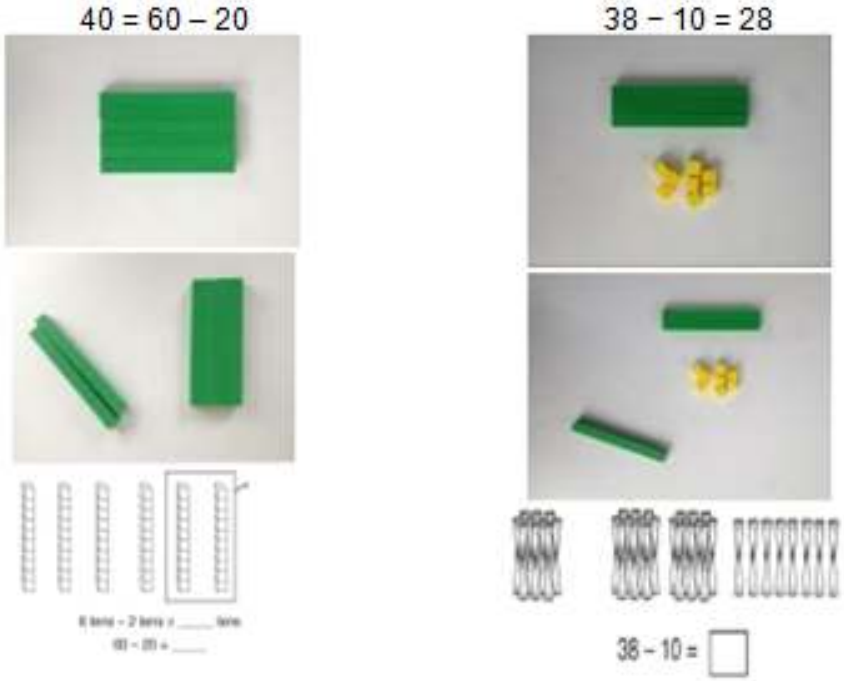
<table>
<tr><th colspan="2">Addition</th></tr>
<tr><td>Joining two groups and then recounting all objects using one-to-one Correspondence
(lots of practice making 10 and numbers to 10 e.g. 6 + 4 = 10 or 3 + 5 = 8)</td><td>3 + 4 = 7
 

 
 
  </td></tr>
<tr><td>Learn number bonds to 20 and demonstrate related facts
Teach addition and subtraction alongside each other as pupils need to see the relationship between the facts.</td><td> 
 
  </td></tr>
<tr><td>Add and subtract one digit numbers and two digit numbers to 20, including zero</td><td>8+1=9
  </td></tr>
<tr><td>Bridging 10
Use ten frames, Singapore bars, egg boxes and number lines to practice.

Chn should start with the larger number and add the smaller number seeing what makes ten and what is left over.</td><td> 
6 + 6 = 12
 
Make 9 in one and 3 in the other. Take one from the 3 to make the 9 into a ten….10+2 = 12</td></tr>
</table>

Subtraction

Taking away should begin with **physical objects**: objects, cubes, Dienes etc	
Subtraction by counting back	
Subtracting a single digit number from a single digit number and a single digit from a two digit by crossing out pictures	
Subtracting using the part part whole (include problem solving with missing digits). ? - 5 = 2	

Subtraction by subtracting from 10 *Children subtract from 10 and not from ones*	14 – 8 = ? 
When subtracting using Dienes children should be taught to regroup a ten rod for 10 ones and then subtract from those ones	 20 – 4 = 16
Subtracting multiples of 10 *Using the vocabulary of 1 ten, 2 tens etc alongside 10, 20, 30 Is very important here as pupils need to understand that it is a 10 not a 1 that is being taken away*	

Multiplication

Counting in multiples of 2, 5 and 10 from zero

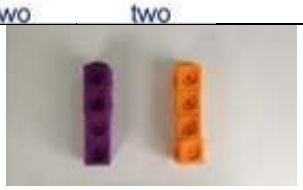
Children should count the number of groups on their fingers as they are skip counting.

2 4 6 8

4 groups of 2 = 8

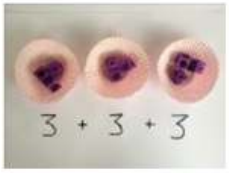
2 × 4 = 8

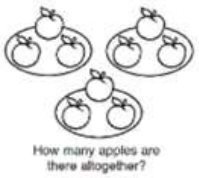
When moving to pictorial/written calculations the vocabulary is important



This image represents two groups of 4 or 4 twice

Solving multiplication problems using repeated addition





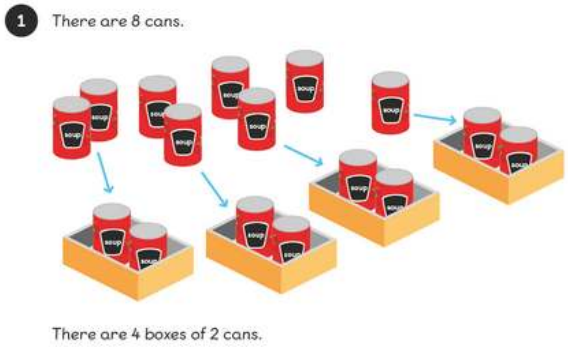
3 + 3 + 3 = 9

Division

Pupils should be taught to divide through working practically and the sharing should be shown below the whole to familiarize children with the concept of the whole.

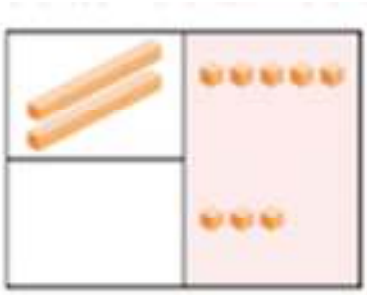
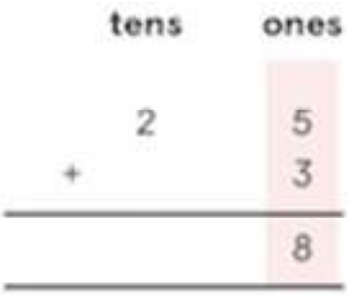
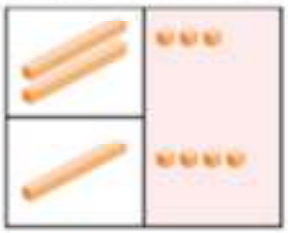
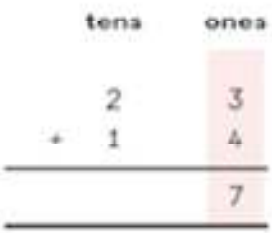
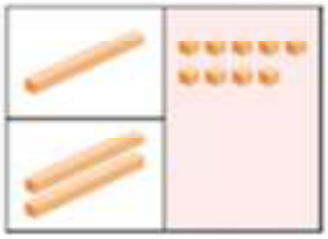
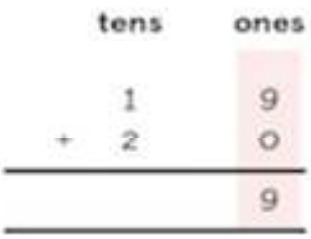
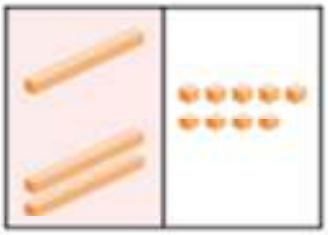
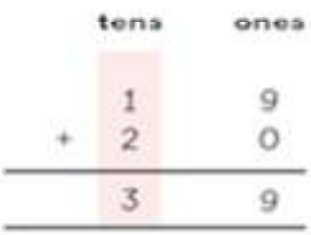
The language of whole and part part should be used.

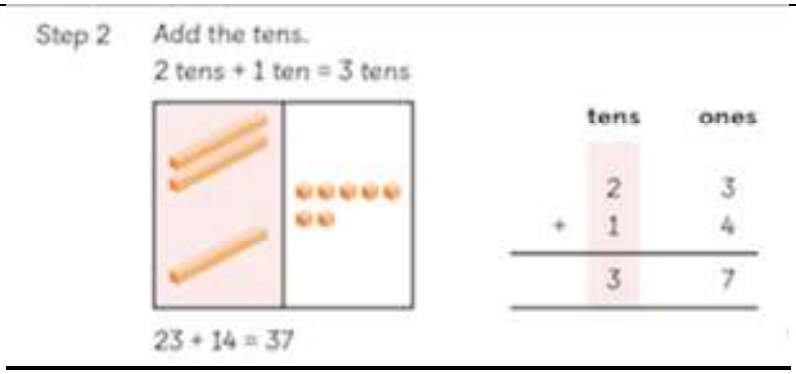
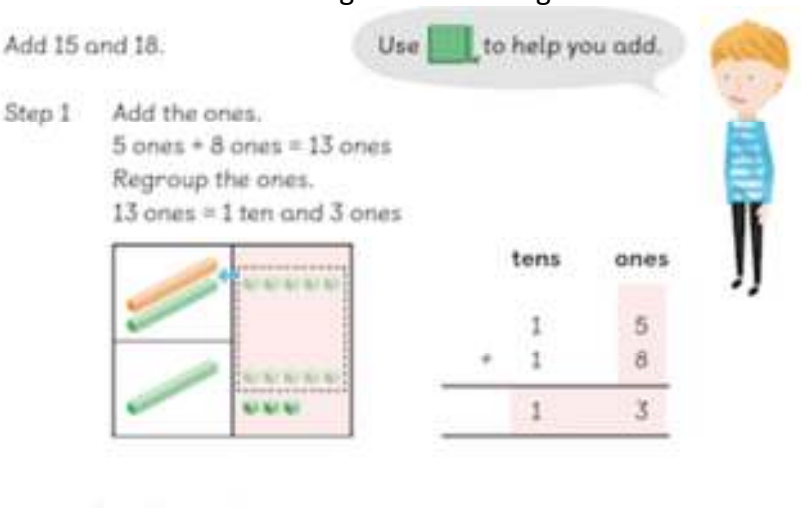
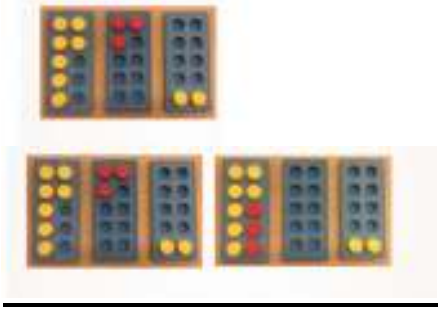
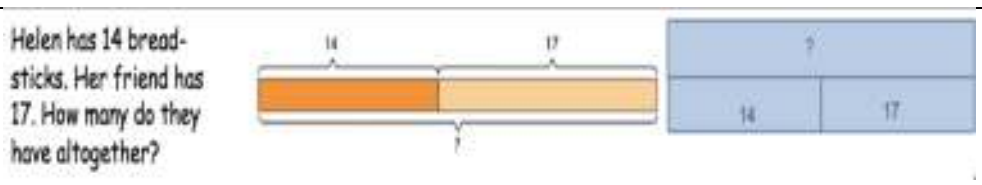
10 ÷ 2 = 5



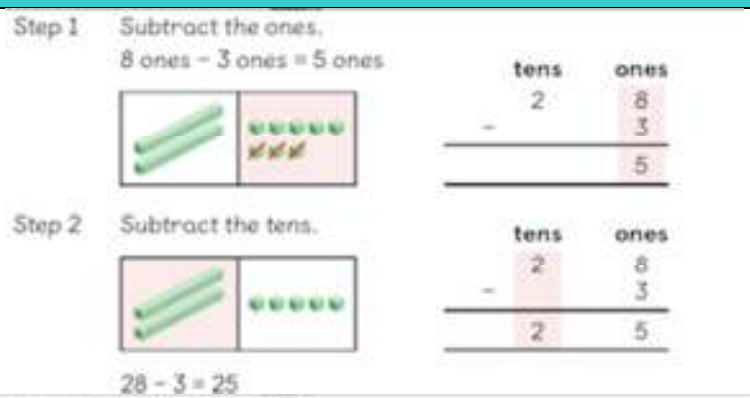
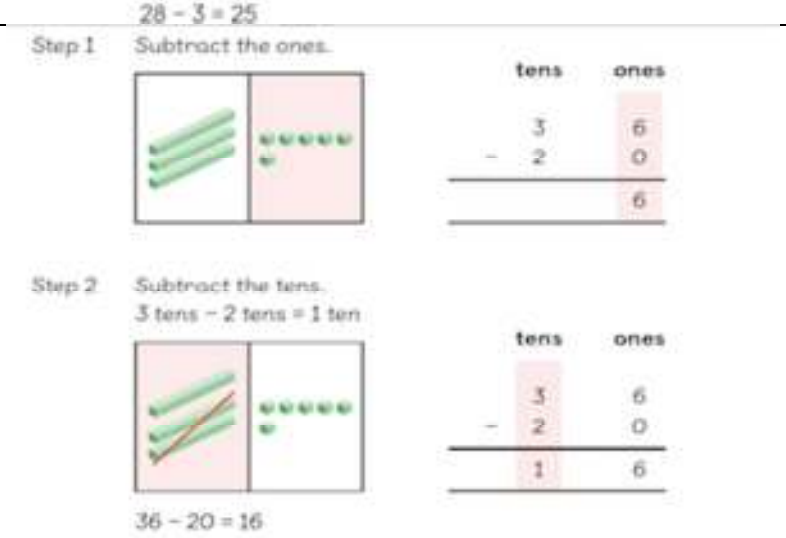
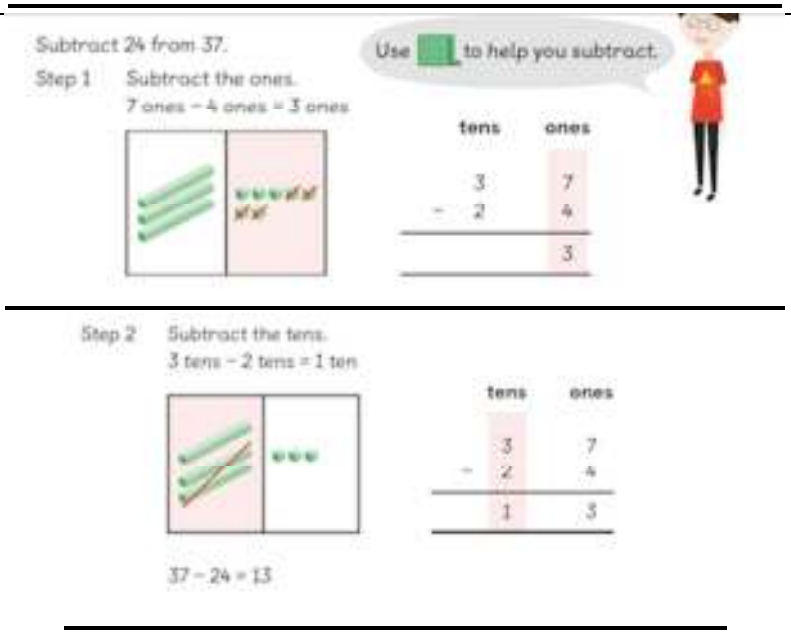
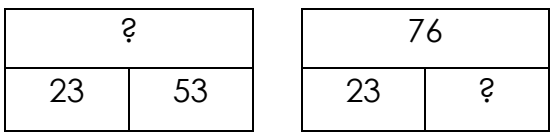
8 ÷ 4 = 2

Year 2

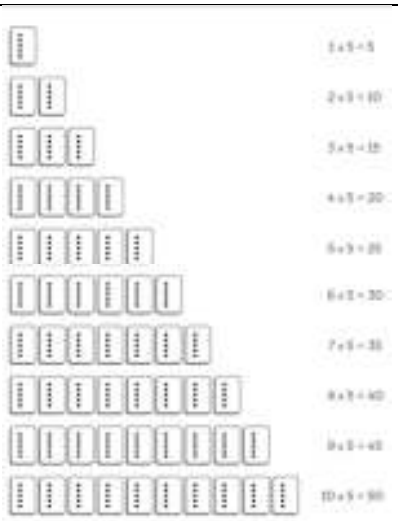
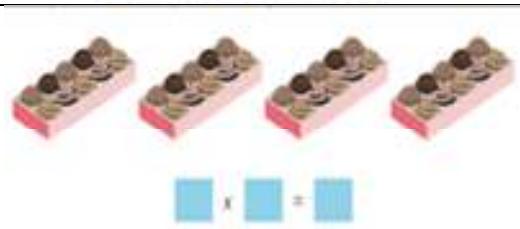
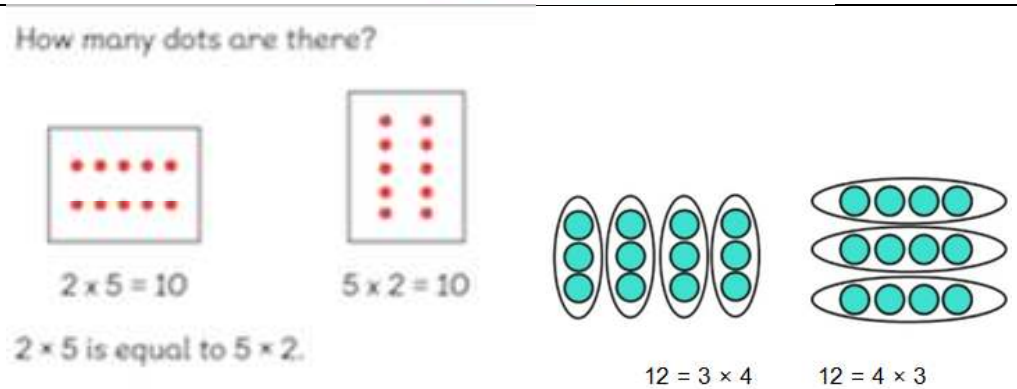
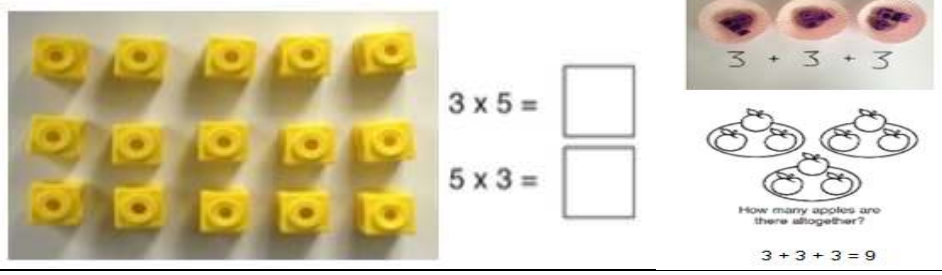
Addition	
Using concrete objects and pictorial representations to add a 2 digit number with a 1 digit number.	 
Using concrete objects and pictorial representations to add a 2 digit number and 10s number.	Step 1 Add the ones. 3 ones + 4 ones = 7 ones   Step 1 Add the ones.   Step 2 Add the tens. 1 ten + 2 tens = 3 tens   19 + 20 = 39

Using concrete objects and pictorial representations to add a 2 2 digit numbers.	 Adding with renaming 
Using concrete objects and pictorial representations to add a 3 single digit numbers.	7+3+2 = leads to 10 + 2 = 
Using the bar to find missing digits. *It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.*	

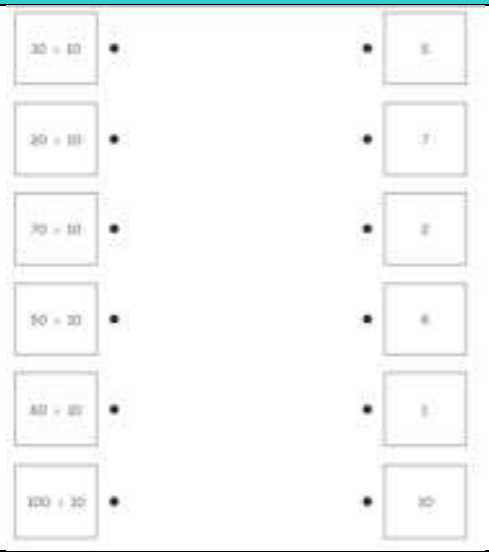
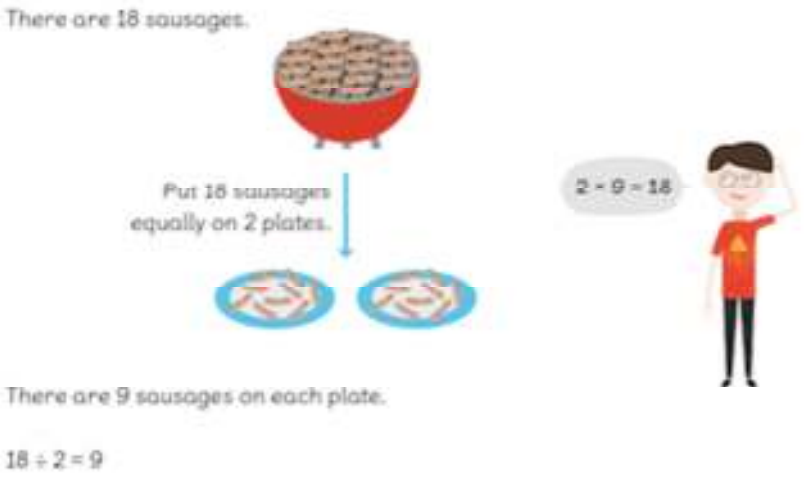
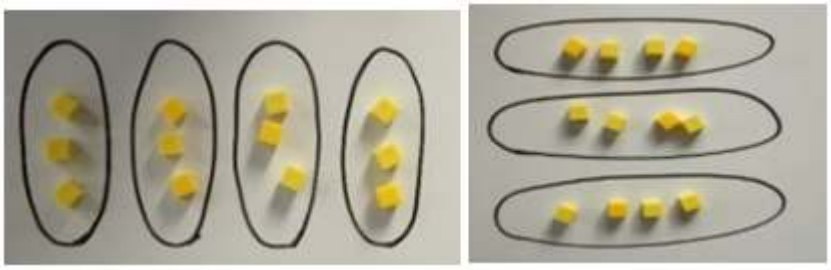
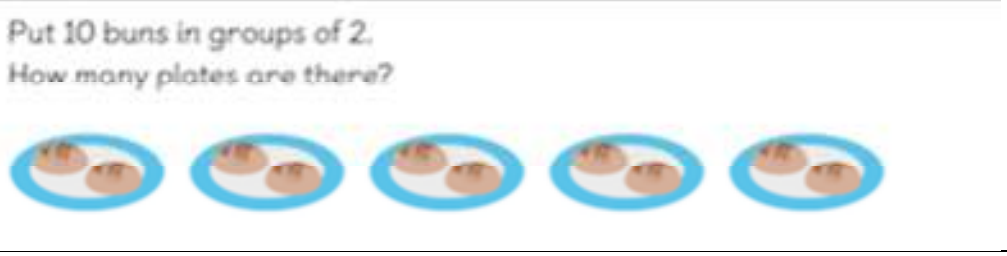
Subtraction

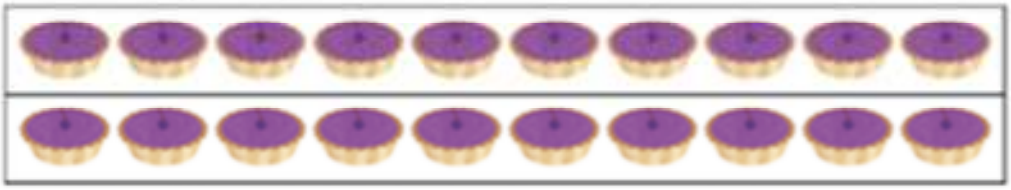
Using concrete objects and pictorial representations to subtract a 1 digit number from 2 digit number.	
Using concrete objects and pictorial representations to subtract a 10s number from 2 digit number.	
Using concrete objects and pictorial representations to subtract a 2 digit number from 2 digit number.	
Recognise and use the inverse relationship between addition and subtraction	 Use this to check calculations and solve missing number problems.

Multiplication

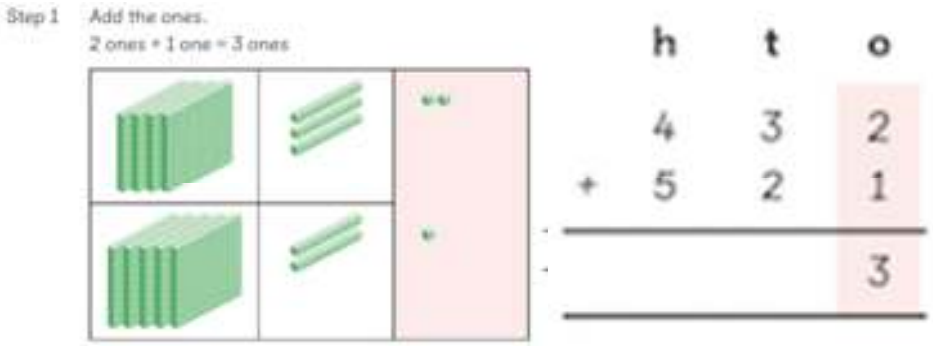
Skip counting in multiples of 2, 3, 5, 10 from 0	
Recall and use multiplication facts for the multiplication tables 2, 5 and 10.	
I can use multiplication (x) and equal (=) sign when writing out my times tables.	
Multiplication is commutative *Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.*	
Solve multiplication problems in context using arrays and repeated addition	

Division

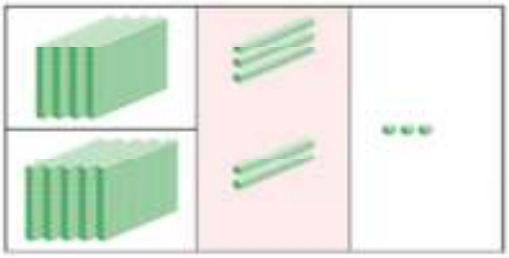
Recall and use division facts for the multiplication tables 2, 5 and 10.	
Solve division problems in context using concrete objects by sharing	
Solve division problems in context using arrays	
I can solve division as grouping.	

	 Put into groups of 5. There are ☐ groups.
I can use the inverse. This should be taught alongside both multiplication and division.	Make a family of multiplication and division facts.  2 x 10 = 20 ——— 20 ÷ 10 = ☐ 10 x 2 = 20 ——— 20 ÷ 2 = ☐

Year 3

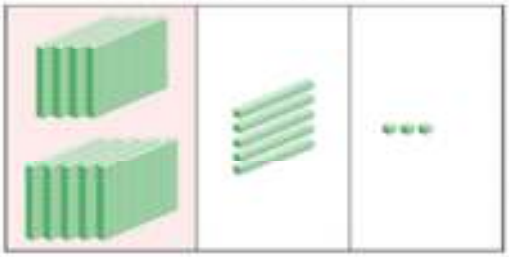
Addition	
Add two three digit numbers. *Children need to use equipment first to support their understanding of place value.* *Children to word gradually to three digit + three digit starting without carrying and gradually moving towards carrying.*	432 + 521 =  Step 1 Add the ones. 2 ones + 1 one = 3 ones h t o 4 3 2 + 5 2 1 3

Step 2 Add the tens.
3 tens + 2 tens = 5 tens



	h	t	o
	4	3	2
+	5	2	1
		5	3

Step 3 Add the hundreds.
4 hundreds + 5 hundreds = 9 hundreds

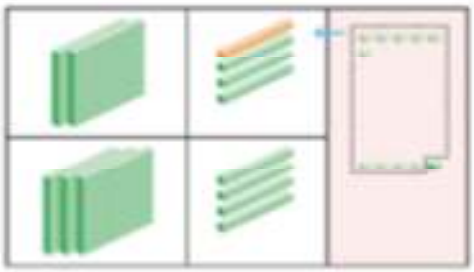


	h	t	o
	4	3	2
+	5	2	1
	9	5	3

432 + 521 = 953

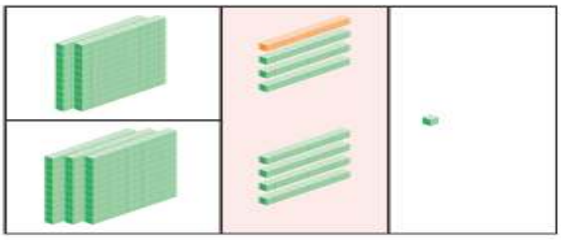
There are 953 flowers altogether.

236 + 345 =



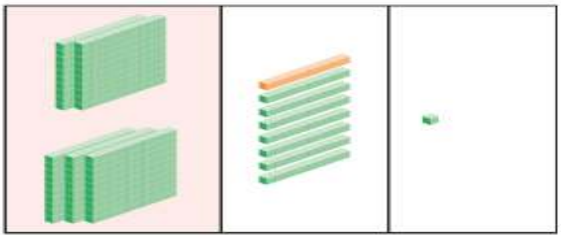
	h	t	o
	2	3	6
+	3	4	5
			1

Step 2 Add the tens.
1 ten + 3 tens + 4 tens = 8 tens



	h	t	o
	2	1 3	6
+	3	4	5
		8	1

Step 3 Add the hundreds.
2 hundreds + 3 hundreds = 5 hundreds

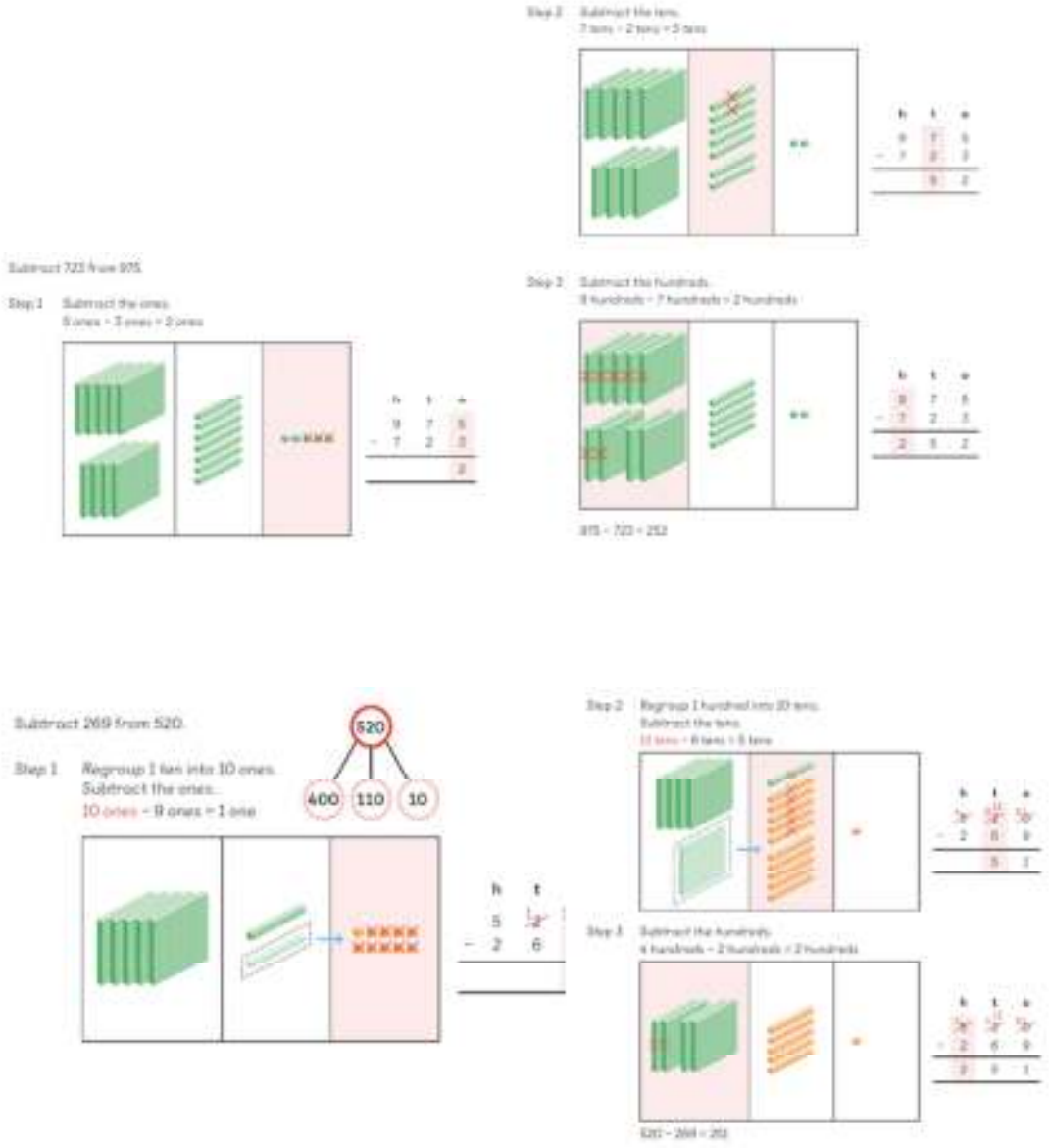


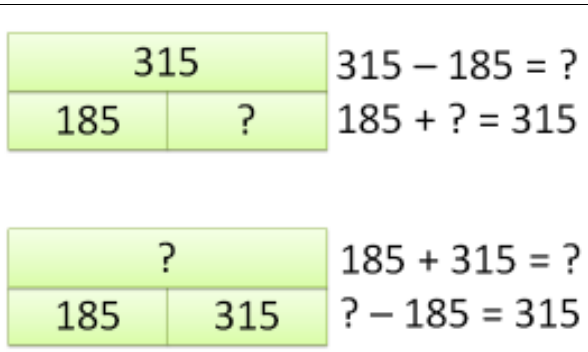
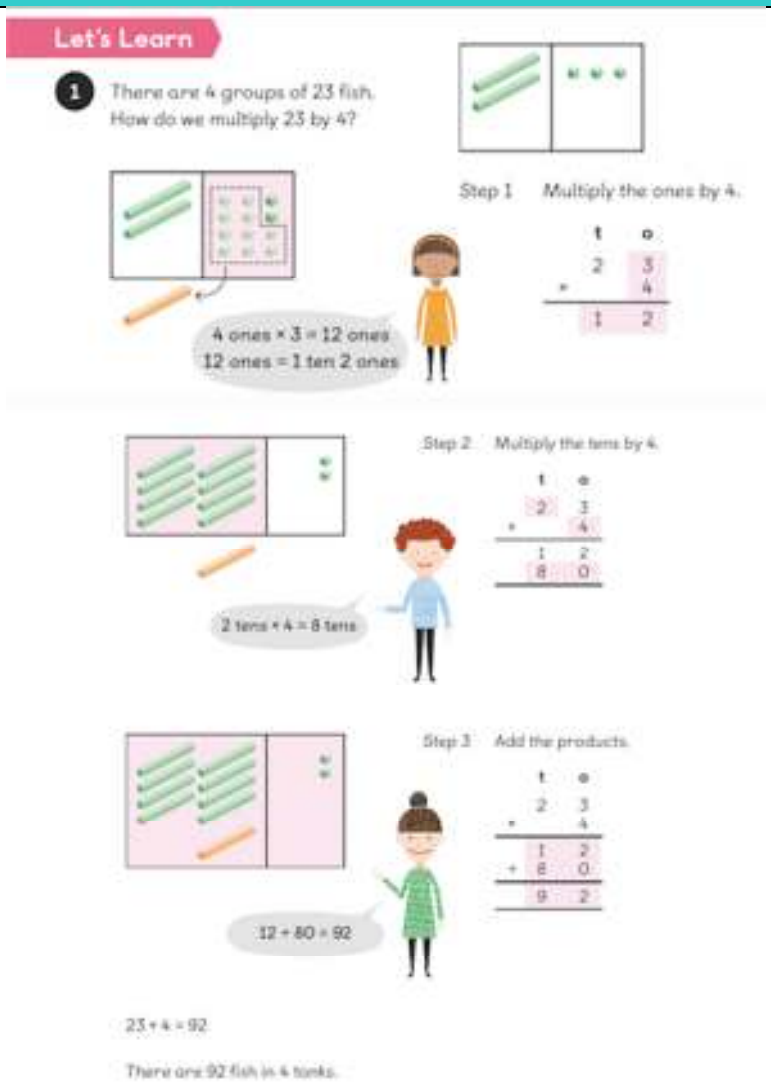
	h	t	o
	2	1 3	6
+	3	4	5
	5	8	1

236 + 345 = 581

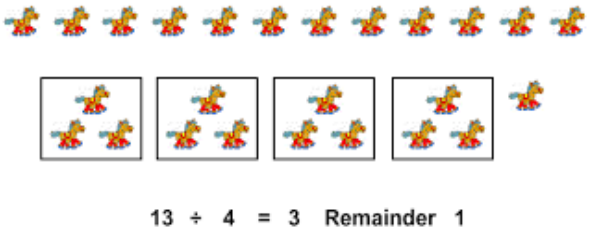
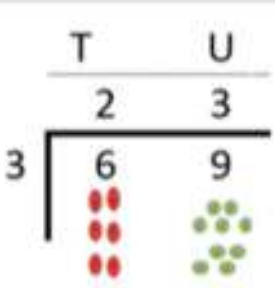
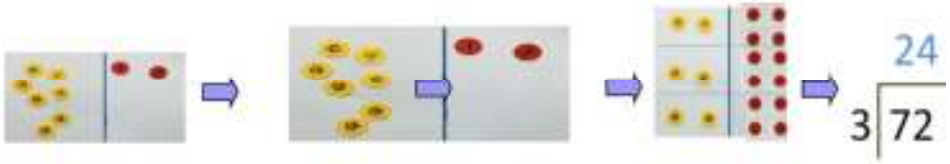
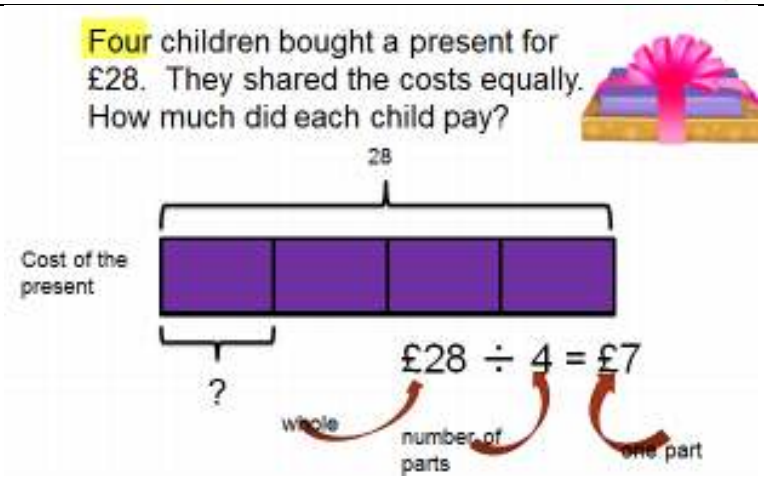
Using the bar to find missing digits. *It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.*	**Bar Model to support understanding of problem solving:** A man sold 230 balloons at a carnival in the morning. He sold another 86 balloons in the evening . How many balloons did he sell in all? 

Subtraction

Subtract up to 3 digits from 3 digits. *Very important for children to use dienes equipment along with a place value chart to support.*	Only when secure with the method should exchanging be introduced. 

Using the bar to find missing digits. *It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.*	
Multiplication	
Children should be able to recall the 2, 5, 10, 3, 4 and 8 times tables. Multiple a two digit number by a one digit.	
Using the bar to solve multiplication problems.	4 children go to the cinema. They each pay £15. How much do they spend altogether? Whole unknown 

Division

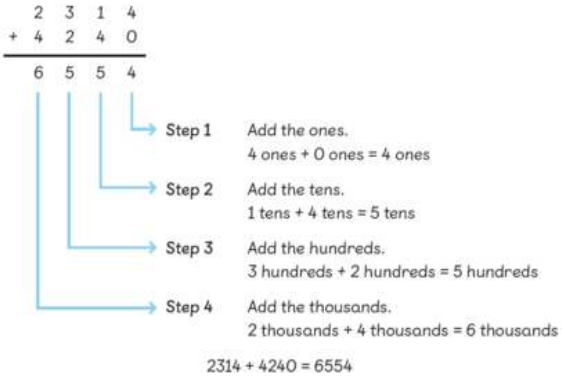
Dividing by grouping undrestanding the concept of remainders.	Start with using the real objects-or objects that represent the calculation.  13 ÷ 4 = 3 Remainder 1
Dividing using short division. *Once children are secure with division as grouping and demonstrate this using number lines, arrays etc., **short division** for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all. Start by introducing the layout of short division by comparing it to an array.*	 **Remind children of correct place value, that 69 is equal to 60 and 9, but in short division, pose:** · *How many 3's in 6? = 2, and record it above the **6 tens**.* · *How many 3's in 9? = 3, and record it above the **9 ones**.* Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. 72÷3), and be taught to 'carry' the remainder onto the next digit. 
Using the bar to aid the solving of division problems.	 Four children bought a present for £28. They shared the costs equally. How much did each child pay? 28 Cost of the present ? £28 ÷ 4 = £7 whole number of parts one part

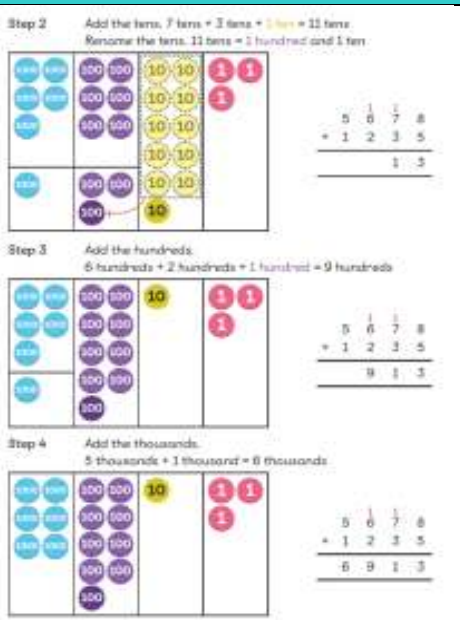
Year 4

Addition

Adding numbers with up to 4 digits.

Again this should start with the children using dienes to support them with lots of discussion about the value of each digit.



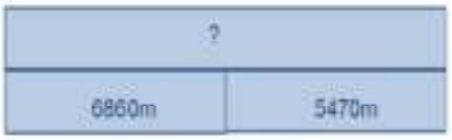


Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.

This is not a form of getting the correct answer but helping to guide children to the correct operation.

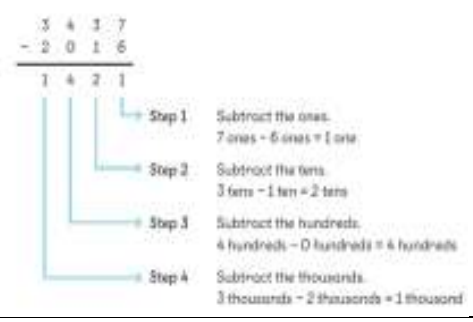
Alison jogs 6,860 metres and Calvin jogs 5,470 metres. How far do they jog altogether?

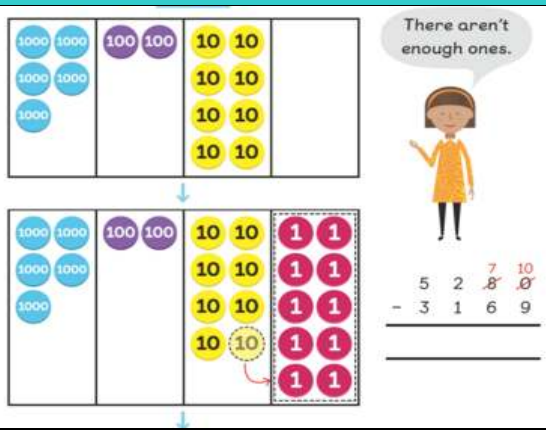


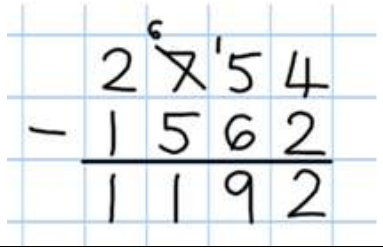
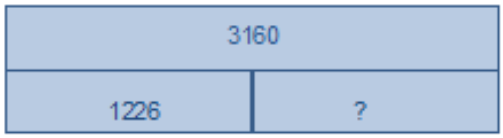
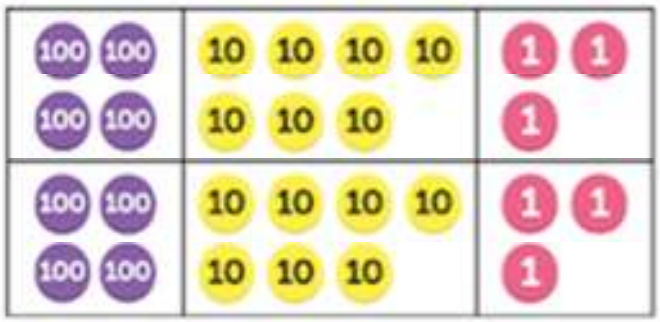
Subtraction

To subtract with numbers up to four digits including exchanging when children are secure.

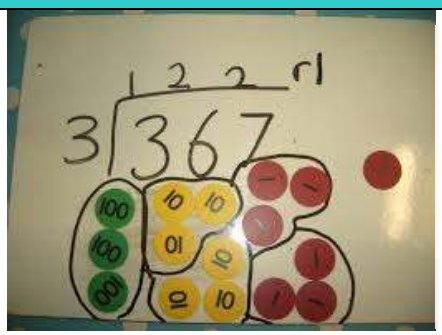
Again children need to use dienes to support their learning.

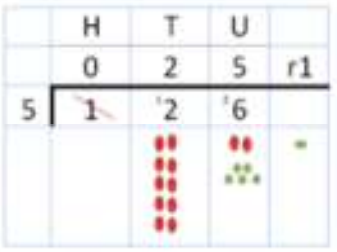


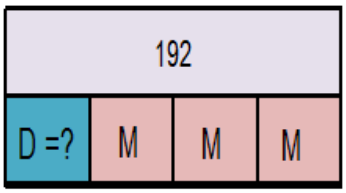


	
Using the bar to find missing digits. *It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.*	There are 3,160 books in a shop. 1,226 are in English and the rest are in French. How many French books are there? 
Multiplication	
Children to know all times tables to 12 x 12. Ladder method to be used with children multiplying both two and three digits by a one digit number.	3 1 4 × 3 1 2 (3 x 4) 3 0 (3 x 10) + 9 0 0 (3 x 300) 9 4 2  
Multiplying using the bar.	A computer costs 5 times as much as a television. The television costs £429. How much does the computer cost? 

<table>
<tr><th colspan="2">Division</th></tr>
<tr><td>Dividing up to three digit numbers by a one digit number using short division.

Only when the children are secure with dividing a two digit number should they move onto a 3 digit number.</td><td> 


</td></tr>
<tr><td>Dividing using the bar.</td><td>Desmond and Melissa collect cards. They have 192 cards in all. Melissa has three times as many cards as Desmond. How many cards does Desmond have?

 
</td></tr>
</table>

Early Years:

Key vocabulary for Addition: add, addition, more than, plus, and, make, balances, altogether, total, sum, equal to, equals, double, most, count on, number line, digit, tens, ones, part-part whole, ten frame

Key vocabulary for Subtraction: minus, subtract, less than, take away, left, one less, count back, equal to, number sentence, part-part whole, ten frame

Key vocabulary for Multiplication: double, equals, groups of, lots of, times, altogether

Key Vocabulary for Division: share, share between, share equally, one each, two each, groups, halve

YEAR 1

Key vocabulary for Addition: add, addition, more than, plus, and, make, balances, altogether, total, sum, equal to, equals, double, most, count on, number line, digit, tens, ones, part-part whole, ten frame, greater/greatest, partition, operation, inverse, number sentence, calculation, number family

Key vocabulary for Subtraction: minus, subtract, less than, take away, left, one less, count back, equal to, number sentence, part-part whole, ten frame leaves, difference between, how many more, how

many fewer / less than, most, least, smaller/smallest, how many left, how much less is?, inverse, calculation, operation, number family

Key vocabulary for Multiplication: double, equals, groups of, lots of, times, altogether multiply, count, array, multiple

Key Vocabulary for Division: share, share between, share equally, one each, two each, group, equal groups of, halve, groups of, lots of, array

YEAR 2

Key vocabulary for Addition: add, addition, more than, plus, and, make, balances, altogether, total, sum, equal to, equals, double, most, count on, number line, digit, tens, ones, part-part whole, ten frame, greater/greatest, partition, operation, inverse, number sentence, calculation, number family, column, tens boundary, bar model

Key vocabulary for Subtraction: minus, subtract, less than, take away, left, one less, count back, equal to, number sentence, part-part whole, ten frame leaves, difference between, how many more, how many fewer / less than, most, least, smaller/smallest, how many left, how much less is?, inverse, calculation, operation, number family, strategy, partition, target ten, bar model

Key vocabulary for Multiplication: double, equals, groups of, lots of, times, altogether multiply, count, array, multiple, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..., bar model

Key Vocabulary for Division: share, share between, share equally, one each, two each, group, equal groups of, halve, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, remainder, bar model

YEAR 3

Key vocabulary for Addition: add, addition, more than, plus, and, make, balances, altogether, total, sum, equal to, equals, double, most, count on, number line, digit, tens, ones, part-part whole, ten frame, partition, operation, inverse, number sentence, calculation, number family, column, tens boundary, bar model, bridging tens boundary, bridging hundreds boundary, increase, vertical, 'carry', expanded, compact,

Key vocabulary for Subtraction: minus, subtract, less than, take away, left, one less, count back, equal to, number sentence, part-part whole, ten frame leaves, difference between, how many more, how many fewer / less than, most, least, how many left, how much less is?, inverse, calculation, operation, number family, strategy, partition, target ten, bar model, exchange, decrease, hundreds, value, digit

Key vocabulary for Multiplication: double, equals, groups of, lots of, times, altogether multiply, count, array, multiple, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, bar model, partition, grid method, multiple, product, tens, units, value, inverse

Key Vocabulary for Division: share, share between, share equally, one each, two each, group, equal groups of, halve, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, remainder, bar model, inverse, short division, 'carry', multiple, chunking, short division,

YEAR 4

Key vocabulary for Addition: add, addition, more than, plus, and, make, balances, altogether, total, sum, equal to, equals, double, most, count on, number line, digit, tens, ones, part-part whole, ten frame, partition, operation, inverse, number sentence, calculation, number family, column, tens boundary, bar model, bridging tens boundary, bridging hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, inverse, decimals, decimal places, decimal point, tenths, hundredths,

Key vocabulary for Subtraction: minus, subtract, less than, take away, left, one less, count back, equal to, number sentence, part-part whole, ten frame leaves, difference between, how many more, how many fewer / less than, most, least, how many left, how much less is?, inverse, calculation, operation, number family, strategy, partition, target ten, bar model, exchange, decrease, hundreds, value, digit

Key vocabulary for Multiplication: double, equals, groups of, lots of, times, altogether multiply, count, array, multiple, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, bar model, partition, grid method, multiple, product, tens, units, value, inverse, inverse

Key Vocabulary for Division: share, share between, share equally, one each, two each, group, equal groups of, halve, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, remainder, bar model, inverse, short division, 'carry', multiple, chunking, short division, divisible by, factor