

Common Calculations Policy



March 2018

Common Calculations Policy

Rationale

This document outlines the progression through calculations taken by pupils within the Cheddar Valley groups of schools: The Wessex Learning Trust and the Mendip Edge Federation. At the heart of this policy lies the understanding that pupils progress through the stages of learning at a rate that is appropriate to them, yet broadly in line with national expectations, and builds upon developing a deep understanding of concepts through rehearsal in the concrete, pictorial and abstract.

Aim

The aim is to promote a journey of learning and recording of mathematics that supports pupils as they transition between settings, to ensure a commonality in language, methodology and visual representations.

	Addition	Subtraction	Multiplication	Division
Yr. 1	<u>Mental Strategies and Vocabulary</u>	<u>Mental Strategies and Vocabulary</u>	<u>Mental Strategies and Vocabulary</u>	<u>Mental Strategies and Vocabulary</u>
Yr. 2	<u>Written Methods</u>	<u>Written Methods</u>	<u>Written Methods</u>	<u>Written Methods</u>
Yr. 3				
Yr. 4	<u>Mental Strategies and Vocabulary</u>	<u>Mental Strategies and Vocabulary</u>	<u>Mental Strategies and Vocabulary</u>	<u>Mental Strategies and Vocabulary</u>
Yr. 5	<u>Written Methods</u>	<u>Written Methods</u>	<u>Written Methods</u>	<u>Written Methods</u>
Yr. 6 +				
	<u>Problem Solving with Bar Models.</u>			

Addition – Mental Strategies and Vocabulary

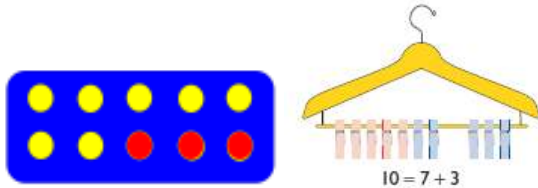
Year 1

Mental Strategies (addition and subtraction)

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions.

They should see addition and subtraction as related operations. E.g. $7 + 3 = 10$ is related to $10 - 3 = 7$, understanding of which could be supported by an image like this.



Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones and develop understanding of place value.

Children have opportunities to explore partitioning numbers in different ways.

e.g. $7 = 6 + 1$, $7 = 5 + 2$, $7 = 4 + 3 =$

Children should begin to understand addition as combining groups and counting on.



Vocabulary

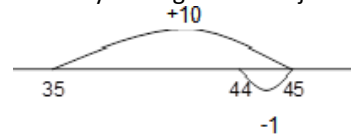
Addition, add, forwards, put together, more than, total, altogether, distance between, difference between, equals = same as, most, pattern, odd, even, digit, counting on.

Year 2

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting forwards in tens from any number should lead to adding multiples of 10.

Number lines should continue to be an important image to support mathematical thinking, for example to model how to add 9 by adding 10 and adjusting.



Children should practise addition to 20 to become increasingly fluent. They should use the facts they know to derive others, e.g. using $7 + 3 = 10$ to find $17 + 3 = 20$, $70 + 30 = 100$. They should use concrete objects such as bead strings and number lines to explore missing numbers – $45 + \underline{\quad} = 50$.

As well as number lines, 100 squares could be used to explore patterns in calculations such as $74 + 11$, $77 + 9$ encouraging children to think about 'What do you notice?' where partitioning or adjusting is used.

Children should learn to check their calculations, by using the inverse.

They should continue to see addition as both combining groups and counting on.

They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g. $23 = 20 + 3 = 10 + 13$.

Vocabulary

+, add, addition, more, plus, make, sum, total, altogether, how many more to make...? how many more is... than...? how much more is...? =, equals, sign, is the same as, Tens, ones, partition Near multiple of 10, tens boundary, More than, one more, two more... ten more... one hundred more

Year 3

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. This will help to develop children's understanding of working mentally.

Children should continue to partition numbers in different ways.

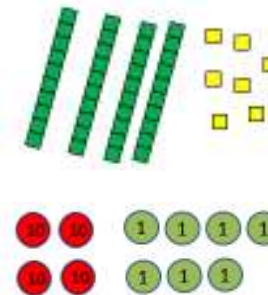
They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g.

Add the nearest multiple of 10, then adjust such as $63 + 29$ is the same as $63 + 30 - 1$;

counting on by partitioning the second number only such as $72 + 31 = 72 + 30 + 1 = 102 + 1 = 103$

Manipulatives can be used to support mental imagery and conceptual understanding. Children need to be shown how these images are related eg.

What's the same? What's different?



Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange

See also Y1 and Y2

Generalisations

- True or false? Addition makes numbers bigger.
- True or false? You can add numbers in any order and still get the same answer.

(Links between addition and subtraction)

When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

$$6 + \square = 11$$

$$6 + 5 = 5 + \square$$

$$6 + 5 = \square + 4$$

$$11 = 5 + \square$$

$$11 = \square + \square + \square$$

These could all be shown pictorially or as part-whole diagrams.

Some Key Questions

How many altogether? How many more to make...? I add ...more. What is the total? How many more is... than...? How much more is...? One more, two more, ten more... What can you see here? Is this true or false? What is the same? What is different?

Generalisation

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd + odd = even; odd + even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.



$$7 + ? = 10$$



Some Key Questions

How many altogether? How many more to make...? How many more is... than...? How much more is...? Is this true or false? If I know that $17 + 2 = 19$, what else do I know? (e.g. $2 + 17 = 19$; $19 - 17 = 2$; $19 - 2 = 17$; $190 - 20 = 170$ etc). What do you notice? What patterns can you see?

Generalisations

Noticing what happens to the digits when you count in tens and hundreds.

Odd + odd = even etc (see Year 2)

Inverses and related facts – develop fluency in finding related addition and subtraction facts.

Develop the knowledge that the inverse relationship can be used as a checking method.

Some Key Questions

What do you notice? What patterns can you see?

When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line?



Addition – Mental Strategies and Vocabulary

Year 4	Year 5	Year 6 +
<p><u>Mental Strategies</u> Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate. Children should continue to partition numbers in different ways.</p> <p>They should be encouraged to choose from a range of strategies:</p> <ul style="list-style-type: none"> Counting forwards and backwards: 124 – 47, count back 40 from 124, then 4 to 80, then 3 to 77 Reordering: 28 + 75, 75 + 28 (thinking of 28 as 25 + 3) Partitioning: counting on or back: 5.6 + 3.7, 5.6 + 3 + 0.7 = 8.6 + 0.7 Partitioning: bridging through multiples of 10: 6070 – 4987, 4987 + 13 + 1000 + 70 Partitioning: compensating – 138 + 69, 138 + 70 - 1 Partitioning: using ‘near’ doubles - 160 + 170 is double 150, then add 10, then add 20, or double 160 and add 10, or double 170 and subtract 10 Partitioning: bridging through 60 to calculate a time interval – What was the time 33 minutes before 2.15pm? Using known facts and place value to find related facts. <p><u>Vocabulary</u> add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.</p>	<p><u>Mental Strategies</u> Children should continue to count regularly, on and back, now including steps of powers of 10. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate. Children should continue to partition numbers in different ways.</p> <p>They should be encouraged to choose from a range of strategies:</p> <ul style="list-style-type: none"> Counting forwards and backwards in tenths and hundredths: 1.7 + 0.55 Reordering: 4.7 + 5.6 – 0.7, 4.7 – 0.7 + 5.6 = 4 + 5.6 Partitioning: counting on or back - 540 + 280, 540 + 200 + 80 Partitioning: bridging through multiples of 10: Partitioning: compensating: 5.7 + 3.9, 5.7 + 4.0 – 0.1 Partitioning: using ‘near’ double: 2.5 + 2.6 is double 2.5 and add 0.1 or double 2.6 and subtract 0.1 Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20? Using known facts and place value to find related facts. <p><u>Vocabulary</u> tens of thousands boundary, Also see previous years</p> <p><u>Generalisation</u> Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9. What do you notice about the differences between consecutive square numbers? Investigate $a - b = (a-1) - (b-1)$ represented visually.</p>	<p><u>Mental Strategies</u> Consolidate previous years.</p> <p>Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. $20 - 5 \times 3 = 5$; $(20 - 5) \times 3 = 45$</p> <p><u>Vocabulary</u> See previous years</p> <p><u>Generalisations</u> Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children should learn acrostic BIDMAS (BODMAS, PEMDAS), or could be encouraged to design their own ways of remembering. Sometimes, always or never true? Subtracting numbers makes them smaller.</p> <p><u>Some Key Questions</u> What do you notice? What’s the same? What’s different? Can you convince me? How do you know?</p>

<p><u>Generalisations</u> Investigate when re-ordering works as a strategy for subtraction. Eg. $20 - 3 - 10 = 20 - 10 - 3$, but $3 - 20 - 10$ would give a different answer.</p> <p><u>Some Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>	<p><u>Some Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>	
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Subtraction – Mental Strategies and Vocabulary

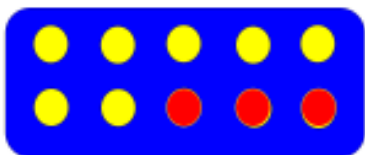
Year 1

Mental Strategies

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions.

They should see addition and subtraction as related operations. E.g. $7 + 3 = 10$ is related to $10 - 3 = 7$, understanding of which could be supported by an image like this.

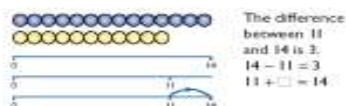


Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones.

Children should begin to understand subtraction as both taking away and finding the difference between, and should find small differences by counting on.



Subtraction as "taking away"



Subtraction as "the difference between"

Vocabulary

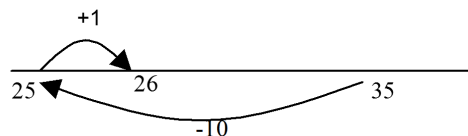
Subtraction, subtract, take away, distance between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit,

Year 2

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting back in tens from any number should lead to subtracting multiples of 10.

Number lines should continue to be an important image to support thinking, for example to model how to subtract 9 by adjusting.



Children should practise subtraction to 20 to become increasingly fluent. They should use the facts they know to derive others, e.g. using $10 - 7 = 3$ and $7 = 10 - 3$ to calculate $100 - 70 = 30$ and $70 = 100 - 30$.

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

As well as number lines, 100 squares could be used to model calculations such as $74 - 11$, $77 - 9$ or $36 - 14$, where partitioning or adjusting are used. On the example above, 1 is in the bottom left corner so that 'up' equates to 'add'.

Children should learn to check their calculations, including by adding to check.

They should continue to see subtraction as both take away and finding the difference, and should find a small difference by counting up.

They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g. $23 = 20 + 3 = 10 + 13$.

Year 3

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.

Children should continue to partition numbers in difference ways.

They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g. counting up (difference, or complementary addition) for $201 - 198$; counting back (taking away / partition into tens and ones) for $201 - 12$.

Calculators can usefully be introduced to encourage fluency by using them for games such as 'Zap' [e.g. Enter the number 567. Can you 'zap' the 6 digit and make the display say 507 by subtracting 1 number?]

The strategy of adjusting can be taken further, e.g. subtract 100 and add one back on to subtract 99. Subtract other near multiples of 10 using this strategy.

Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange

See also Y1 and Y2

Generalisations

Noticing what happens to the digits when you count in tens and hundreds.

Odd – odd = even etc (see Year 2)

Inverses and related facts – develop fluency in finding related addition and subtraction facts.

Develop the knowledge that the inverse relationship can be used as a checking method.

Generalisations

- True or false? Subtraction makes numbers smaller
- When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

Children could see the image below and consider, "What can you see here?" e.g.

3 yellow, 1 red, 1 blue. $3 + 1 + 1 = 5$

2 circles, 2 triangles, 1

square. $2 + 2 + 1 = 5$

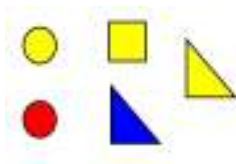
I see 2 shapes with

curved lines and 3 with

straight lines. $5 = 2 + 3$

$5 = 3 + 1 + 1 = 2 + 2 + 1 =$

$2 + 3$



Some Key Questions

How many more to make...? How many more is... than...?

How much more is...? How many are left/left over? How

many have gone? One less, two less, ten less... How

many fewer is... than...? How much less is...?

What can you see here?

Is this true or false?

Vocabulary

Subtraction, subtract, take away, difference, difference between, minus

Tens, ones, partition

Near multiple of 10, tens boundary

Less than, one less, two less... ten less... one hundred less

More, one more, two more... ten more... one hundred more

Generalisations

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd – odd = even; odd – even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.



$$15 + 5 = 20$$

Some Key Questions

How many more to make...? How many more is... than...? How

much more is...? How many are left/left over? How many

fewer is... than...? How much less is...?

Is this true or false?

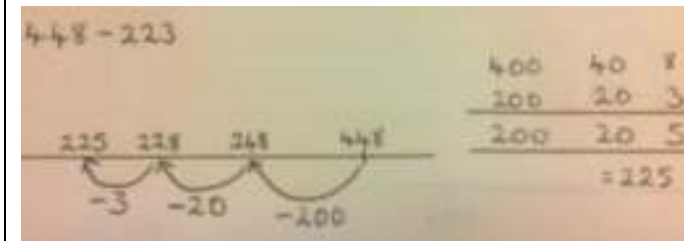
If I know that $7 + 2 = 9$, what else do I know? (e.g. $2 + 7 = 9$; $9 - 7 = 2$; $9 - 2 = 7$; $90 - 20 = 70$ etc).

What do you notice? What patterns can you see?

Key Questions

What do you notice? What patterns can you see?

When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line




Subtraction – Mental Strategies and Vocabulary

Year 4	Year 5	Year 6 +
<p><u>Mental Strategies</u> Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate. Children should continue to partition numbers in different ways.</p> <p>They should be encouraged to choose from a range of strategies:</p> <ul style="list-style-type: none"> Counting forwards and backwards: $124 - 47$, count back 40 from 124, then 4 to 80, then 3 to 77 Reordering: $28 + 75$, $75 + 28$ (thinking of 28 as $25 + 3$) Partitioning: counting on or back: $5.6 + 3.7$, $5.6 + 3 + 0.7 = 8.6 + 0.7$ Partitioning: bridging through multiples of 10: $6070 - 4987$, $4987 + 13 + 1000 + 70$ Partitioning: compensating – $138 + 69$, $138 + 70 - 1$ Partitioning: using 'near' doubles – $160 + 170$ is double 150, then add 10, then add 20, or double 160 and add 10, or double 170 and subtract 10 Partitioning: bridging through 60 to calculate a time interval – What was the time 33 minutes before 2.15pm? Using known facts and place value to find related facts. <p><u>Vocabulary</u> add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.</p>	<p><u>Mental Strategies</u> Children should continue to count regularly, on and back, now including steps of powers of 10. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate. Children should continue to partition numbers in different ways.</p> <p>They should be encouraged to choose from a range of strategies:</p> <ul style="list-style-type: none"> Counting forwards and backwards in tenths and hundredths: $1.7 + 0.55$ Reordering: $4.7 + 5.6 - 0.7$, $4.7 - 0.7 + 5.6 = 4 + 5.6$ Partitioning: counting on or back – $540 + 280$, $540 + 200 + 80$ Partitioning: bridging through multiples of 10: Partitioning: compensating: $5.7 + 3.9$, $5.7 + 4.0 - 0.1$ Partitioning: using 'near' double: $2.5 + 2.6$ is double 2.5 and add 0.1 or double 2.6 and subtract 0.1 Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20? Using known facts and place value to find related facts. <p><u>Vocabulary</u> tens of thousands boundary, Also see previous years</p> <p><u>Generalisations</u> Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9. What do you notice about the differences between consecutive square numbers? Investigate $a - b = (a-1) - (b-1)$ represented visually.</p>	<p><u>Mental Strategies</u> Consolidate previous years.</p> <p>Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. $20 - 5 \times 3 = 5$; $(20 - 5) \times 3 = 45$</p> <p><u>Vocabulary</u> See previous years</p> <p><u>Generalisations</u> Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering. Sometimes, always or never true? Subtracting numbers makes them smaller.</p> <p><u>Some Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>

<p><u>Generalisations</u> Investigate when re-ordering works as a strategy for subtraction. Eg. $20 - 3 - 10 = 20 - 10 - 3$, but $3 - 20 - 10$ would give a different answer.</p> <p><u>Some Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>	<p><u>Some Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>	
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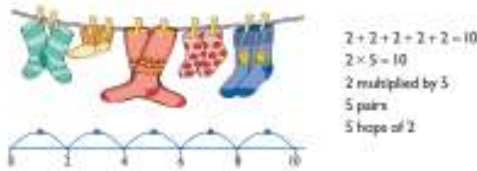






Multiplication – Mental Strategies and Vocabulary

Year 1	Year 2	Year 3
<p><u>Mental Strategies</u> Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10. Children should memorise and reason with numbers in 2, 5 and 10 times tables They should see ways to represent odd and even numbers. This will help them to understand the pattern in numbers.</p>  <p>Children should begin to understand multiplication as scaling in terms of double and half. (e.g. that tower of cubes is double the height of the other tower)</p> <p><u>Vocabulary</u> Ones, groups, lots of, doubling repeated addition groups of, lots of, times, columns, rows longer, bigger, higher etc times as (big, long, wide ...etc)</p> <p><u>Generalisations</u> Understand 6 counters can be arranged as 3+3 or 2+2+2</p> <p>Understand that when counting in twos, the numbers are always even.</p> <p><u>Some Key Questions</u> Why is an even number an even number? What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>	<p><u>Mental Strategies</u> Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Number lines should continue to be an important image to support thinking, for example</p> <p>Children should practise times table facts $2 \times 1 =$ $2 \times 2 =$ $2 \times 3 =$</p> <p>Use a clock face to support understanding of counting in 5s. Use money to support counting in 2s, 5s, 10s, 20s, 50s</p> <p><u>Vocabulary</u> multiple, multiplication array, multiplication tables / facts groups of, lots of, times, columns, rows</p> <p><u>Generalisations</u> Commutative law shown on array (video)</p> <p>Repeated addition can be shown mentally on a number line</p> <p>Inverse relationship between multiplication and division. Use an array to explore how numbers can be organised into groups.</p> <p><u>Some Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>	<p><u>Mental Strategies</u> Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10. The number line should continue to be used as an important image to support thinking, and the use of informal jottings and drawings to solve problems should be encouraged.</p> <p>Children should practise times table facts $3 \times 1 =$ $3 \times 2 =$ $3 \times 3 =$</p> <p><u>Vocabulary</u> partition inverse</p> <p><u>Generalisations</u> Connecting x2, x4 and x8 through multiplication facts</p> <p>Comparing times tables with the same times tables which is ten times bigger. If $4 \times 3 = 12$, then we know $4 \times 30 = 120$. Use place value counters to demonstrate this.</p> <p>When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)</p> <p><u>Some Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>

Multiplication – Mental Strategies and Vocabulary

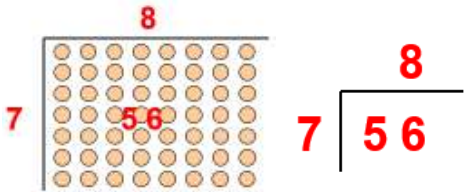
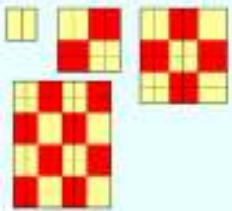
Year 4	Year 5	Year 6 +
<p><u>Mental Strategies</u> Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100. Become fluent and confident to recall all tables to $\times 12$ Use the context of a week and a calendar to support the 7 times table (e.g. how many days in 5 weeks?) Use of finger strategy for 9 times table.</p> <p>Multiply 3 numbers together The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. They should be encouraged to choose from a range of strategies:</p> <ul style="list-style-type: none"> - Partitioning using $\times 10$, $\times 20$ etc - Doubling to solve $\times 2$, $\times 4$, $\times 8$ - Recall of times tables - Use of commutativity of multiplication <p><u>Vocabulary</u> Factor</p> <p><u>Generalisations</u> Children given the opportunity to investigate numbers multiplied by 1 and 0.</p> <p>When they know multiplication facts up to $\times 12$, do they know what $\times 13$ is? (i.e. can they use 4×12 to work out 4×13 and 4×14 and beyond?)</p> <p><u>Some Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>	<p><u>Mental Strategies</u> Children should continue to count regularly, on and back, now including steps of powers of 10. Multiply by 10, 100, 1000, including decimals (Moving Digits ITP) The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. They should be encouraged to choose from a range of strategies to solve problems mentally:</p> <ul style="list-style-type: none"> - Partitioning using $\times 10$, $\times 20$ etc - Doubling to solve $\times 2$, $\times 4$, $\times 8$ - Recall of times tables - Use of commutativity of multiplication <p>If children know the times table facts to 12×12. Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table)</p> <p><u>Vocabulary</u> cube numbers prime numbers square numbers common factors prime number, prime factors composite numbers</p> <p><u>Generalisations</u> Relating arrays to an understanding of square numbers and making cubes to show cube numbers. Understanding that the use of scaling by multiples of 10 can be used to convert between units of measure (e.g. metres to kilometres means to times by 1000)</p> <p><u>Some Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know? How do you know this is a prime number?</p>	<p><u>Mental Strategies</u> Consolidate previous years.</p> <p>Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. $20 - 5 \times 3 = 5$; $(20 - 5) \times 3 = 45$</p> <p>They should be encouraged to choose from a range of strategies to solve problems mentally:</p> <ul style="list-style-type: none"> - Partitioning using $\times 10$, $\times 20$ etc - Doubling to solve $\times 2$, $\times 4$, $\times 8$ - Recall of times tables - Use of commutativity of multiplication <p>If children know the times table facts to 12×12. Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table)</p> <p><u>Vocabulary</u> See previous years common factor product</p> <p><u>Generalisations</u> Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering. Understanding the use of multiplication to support conversions between units of measurement.</p> <p><u>Some Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know?</p>

Division – Mental Strategies and Vocabulary

Year 1	Year 2	Year 3
<p><u>Mental Strategies</u> Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.</p> <p>They should begin to recognise the number of groups counted to support understanding of relationship between multiplication and division.</p>  <p>Children should begin to understand division as both sharing and grouping.</p> <p>Sharing – 6 sweets are shared between 2 people. How many do they have each?</p>  <p>Grouping- How many 2's are in 6?</p>  <p>They should use objects to group and share amounts to develop understanding of division in a practical sense. E.g. using Numicon to find out how many 5's are in 30? How many pairs of gloves if you have 12 gloves?</p> <p>Children should begin to explore finding simple fractions of objects, numbers and quantities.</p> <p>E.g. 16 children went to the park at the weekend. Half that number went swimming. How many children went swimming?</p>	<p><u>Mental Strategies</u> Children should count regularly, on and back, in steps of 2, 3, 5 and 10.</p> <p>Children who are able to count in twos, threes, fives and tens can use this knowledge to work out other facts such as 2×6, 5×4, 10×9. Show the children how to hold out their fingers and count, touching each finger in turn. So for 2×6 (six twos), hold up 6 fingers:</p>  <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Touching the fingers in turn is a means of keeping track of how far the children have gone in creating a sequence of numbers. The physical action can later be visualised without any actual movement</p> </div> <p>This can then be used to support finding out 'How many 3's are in 18?' and children count along fingers in 3's therefore making link between multiplication and division.</p> <p>Children should continue to develop understanding of division as sharing and grouping.</p>  <p>15 pencils shared between 3 pots, how many in each pot?</p> <p>Children should be given opportunities to find a half, a quarter and a third of shapes, objects, numbers and quantities. Finding a fraction of a number of objects to be related to sharing.</p> <p>They will explore visually and understand how some fractions are equivalent – e.g. two quarters is the same as one half.</p> <p>Use children's intuition to support understanding of fractions as an answer to a sharing problem.</p> <p>3 apples shared between 4 people = $\frac{3}{4}$</p> 	<p><u>Mental Strategies</u> Children should count regularly, on and back, in steps of 3, 4 and 8. Children are encouraged to use what they know about known times table facts to work out other times tables. This then helps them to make new connections (e.g. through doubling they make connections between the 2, 4 and 8 times tables).</p> <p>Children will make use multiplication and division facts they know to make links with other facts.</p> <p>$3 \times 2 = 6$, $6 \div 3 = 2$, $2 = 6 \div 3$ $30 \times 2 = 60$, $60 \div 3 = 20$, $2 = 60 \div 30$</p> <p>They should be given opportunities to solve grouping and sharing problems practically (including where there is a remainder but the answer needs to be given as a whole number) e.g. Pencils are sold in packs of 10. How many packs will I need to buy for 24 children?</p> <p>Children should be given the opportunity to further develop understanding of division (sharing) to be used to find a fraction of a quantity or measure.</p> <p>Use children's intuition to support understanding of fractions as an answer to a sharing problem.</p> <p>3 apples shared between 4 people = $\frac{3}{4}$</p>  <p><u>Vocabulary</u> See Y1 and Y2 inverse</p> <p><u>Generalisations</u> Inverses and related facts – develop fluency in finding related multiplication and division facts. Develop the knowledge that the inverse relationship can be used as a checking method.</p>

<p><u>Vocabulary</u> share, share equally, one each, two each..., group, groups of, lots of, array</p> <p><u>Generalisations</u></p> <ul style="list-style-type: none"> • True or false? I can only halve even numbers. • Grouping and sharing are different types of problems. Some problems need solving by grouping and some by sharing. Encourage children to practically work out which they are doing. <p><u>Some Key Questions</u> How many groups of...? How many in each group? Share... equally into... What can do you notice?</p>	<p><u>Vocabulary</u> group in pairs, 3s ... 10s etc equal groups of divide, ÷, divided by, divided into, remainder</p> <p><u>Generalisations</u> Noticing how counting in multiples of 2, 5 and 10 relates to the number of groups you have counted (introducing times tables)</p> <p>An understanding of the more you share between, the less each person will get (e.g. would you prefer to share these grapes between 2 people or 3 people? Why?)</p> <p>Secure understanding of grouping means you count the number of groups you have made. Whereas sharing means you count the number of objects in each group.</p> <p><u>Some Key Questions</u> How many 10s can you subtract from 60? I think of a number and double it. My answer is 8. What was my number? If $12 \times 2 = 24$, what is $24 \div 2$? Questions in the context of money and measures (e.g. how many 10p coins do I need to have 60p? How many 100ml cups will I need to reach 600ml?)</p>	<p><u>Some Key Questions</u> Questions in the context of money and measures that involve remainders (e.g. How many lengths of 10cm can I cut from 81cm of string? You have £54. How many £10 teddies can you buy?) What is the missing number? $17 = 5 \times 3 + \underline{\quad}$ $\underline{\quad} = 2 \times 8 + 1$</p>
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Division – Mental Strategies and Vocabulary

Year 4	Year 5	Year 6 +
<p><u>Mental Strategies</u> Children should experience regular counting on and back from different numbers in multiples of 6, 7, 9, 25 and 1000. Children should learn the multiplication facts to 12 x 12.</p> <p><u>Vocabulary</u> see years 1-3 divide, divided by, divisible by, divided into share between, groups of factor, factor pair, multiple times as (big, long, wide ...etc) equals, remainder, quotient, divisor inverse</p> <p><u>Towards a formal written method</u> Alongside pictorial representations and the use of models and images, children should progress onto short division using a bus stop method.</p>  <p>Place value counters can be used to support children apply their knowledge of grouping. Reference should be made to the value of each digit in the dividend.</p> <p><u>Each digit as a multiple of the divisor</u> 'How many groups of 3 are there in the hundreds column?' 'How many groups of 3 are there in the tens column?' 'How many groups of 3 are there in the units/ones column?'</p>	<p><u>Mental Strategies</u> Children should count regularly using a range of multiples, and powers of 10, 100 and 1000, building fluency. Children should practice and apply the multiplication facts to 12 x 12.</p> <p><u>Vocabulary</u> see year 4 common factors prime number, prime factors composite numbers short division square number cube number inverse power of</p> <p><u>Generalisations</u> The = sign means equality. Take it in turn to change one side of this equation, using multiplication and division, e.g. Start: $24 = 24$ Player 1: $4 \times 6 = 24$ Player 2: $4 \times 6 = 12 \times 2$ Player 1: $48 \div 2 = 12 \times 2$</p> <p>Sometimes, always, never true questions about multiples and divisibility. E.g.:</p> <ul style="list-style-type: none"> If the last two digits of a number are divisible by 4, the number will be divisible by 4. If the digital root of a number is 9, the number will be divisible by 9. When you square an even number the result will be divisible by 4 (one example of 'proof' shown left) 	<p><u>Mental Strategies</u> Children should count regularly, building on previous work in previous years. Children should practice and apply the multiplication facts to 12 x 12.</p> <p><u>Vocabulary</u> see years 4 and 5</p> <p><u>Generalisations</u> Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.</p> <p>Sometimes, always, never true questions about multiples and divisibility. E.g.: If a number is divisible by 3 and 4, it will also be divisible by 12. (also see year 4 and 5, and the hyperlink from the Y5 column)</p> <p>Using what you know about rules of divisibility, do you think 7919 is a prime number? Explain your answer.</p> <div data-bbox="1545 1002 2002 1347"> <p><u>Some Key Questions for Year 4 to 6+</u></p> <p>What do you notice?</p> <p>What's the same? What's different?</p> <p>Can you convince me?</p> <p>How do you know?</p> </div>

$$\begin{array}{r} 112 \\ 3 \overline{) 336} \\ \underline{336} \\ 0 \end{array}$$



When children have conceptual understanding and fluency using the bus stop method without remainders, they can then progress onto 'carrying' their remainder across to the next digit.

Generalisations

True or false? Dividing by 10 is the same as dividing by 2 and then dividing by 5. Can you find any more rules like this?

Is it sometimes, always or never true that $\square \div \Delta = \Delta \div \square$?

Inverses and deriving facts. 'Know one, get lots free!' e.g.: $2 \times 3 = 6$, so $3 \times 2 = 6$, $6 \div 2 = 3$, $60 \div 20 = 3$, $600 \div 3 = 200$ etc.


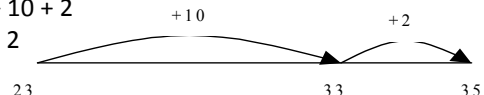
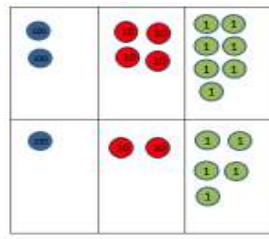
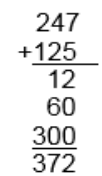
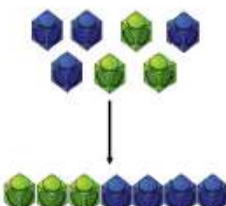
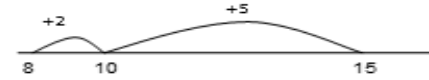
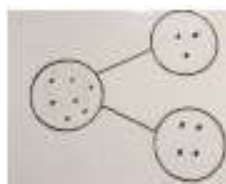
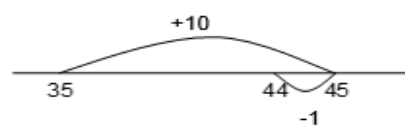

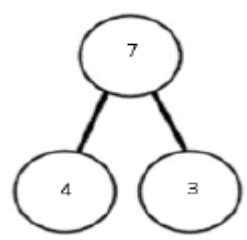
Sometimes, always, never true questions about multiples and divisibility. (When looking at the examples on this page, remember that they **may not** be 'always true'!) E.g.:

- Multiples of 5 end in 0 or 5.
- The digital root of a multiple of 3 will be 3, 6 or 9.
- The sum of 4 even numbers is divisible by 4.



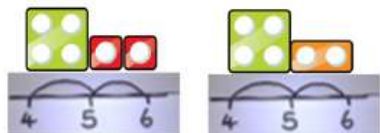
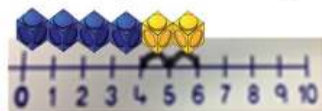
Written Methods of Addition



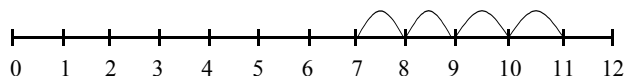
Objectives	Year 1	Guidance	Objectives	Year 2	Guidance	Objectives	Year 3	Guidance
	<p>Counting and Combining sets of Objects</p> <p>Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation)</p> 			<p>Missing number problems e.g. $14 + 5 - 10 + \quad$, $32 + \quad + \quad = 100$, $35 = 1 + \quad + 5$</p> <p>It is valuable to use a range of representations (also see Y1). Continue to use number lines to develop understanding of: <u>Counting on in tens and ones</u></p> $23 + 12 = 23 + 10 + 2$ $= 33 + 2$ $= 35$ 		<p>Missing number problems using a range of equations as in Y1 and Y2 but with appropriate larger numbers.</p> <p>Partition into tens and ones</p> <p>Partition both numbers and recombine. Count on by partitioning the second number only e.g.</p> $247 + 125 = 247 + 100 + 20 + 5$ $= 347 + 20 + 5$ $= 367 + 5$ $= 372$ <p>Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.</p> <p>Towards a formal written method</p> <p>Introduce expanded column addition modelled with concrete resources e.g. place value counters. (Dienes could be used for those who need a less abstract representation)</p>  $200 + 40 + 7$ $100 + 20 + 5$ $300 + 60 + 12 = 372$ 		
	<p>Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).</p> 			<p><u>Partitioning and bridging through 10</u></p> <p>The steps in addition often bridge through a multiple of 10 e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.</p> $8 + 7 = 15$ 				
	<p>This could be recorded pictorially using the part-part-whole model.</p> 			<p><u>Adding 9 or 11 by adding 10 and adjusting by 1</u></p> $35 + 9 = 44$ <p>(add 10 take 1)</p> 				
	<p>Understanding of counting on with a number stick.</p> 			<p>Towards a written Method</p> <p><u>Developing use of the part-part-whole model</u></p> 				
				$\overline{4} + 3 = 7 \quad 7 = 4 + 3$				

Understanding of counting on with a number line.
(supported by models and images).

Counting on using number lines using cubes or Numicon.



$$7 + 4$$



+ = Signs and missing numbers

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

$$2 = 1 + 1$$

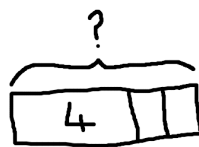
$$2 + 3 = 4 + 1$$

Missing numbers need to be placed in all possible places.

$$3 + 4 = \quad = 3 + 4$$

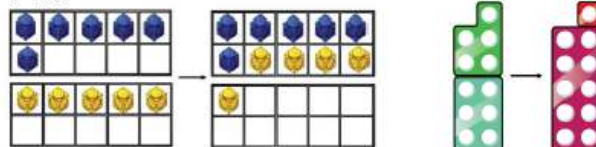
$$3 + \quad = 7 \quad 7 = \quad + 4$$

Using the bar model to encourage children to count on



Regrouping to make 10; using ten frames and counters/cubes or using Numicon.

$$6 + 5$$



Partitioning in different ways and recombine

$$47 + 25$$



Leading to exchanging:

$$72$$



Expanded written method

$$47 + 25$$

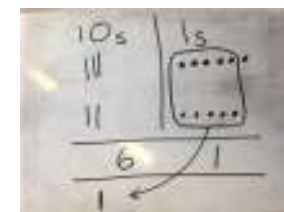
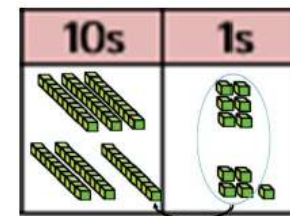
$$40 + 7 + 20 + 5 =$$

$$40 + 20 + 7 + 5 =$$

$$60 + 12 = 72$$

$$\begin{array}{r} 40 + 7 \\ + 20 + 5 \\ \hline 60 + 12 = 72 \end{array}$$

$$36 + 25$$



Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method. Children should carry the ten below the line.

$$\begin{array}{r} 35 \\ + 24 \\ \hline 59 \end{array}$$

When carrying from the ones to tens (or tens to hundreds) some children may find it necessary to reinforce the value of what is being carried by showing it as a multiple of 10 (or 100) below the line but maintaining the place value.

$$\begin{array}{r} 247 \\ + 125 \\ \hline 372 \\ 10 \end{array} \quad \begin{array}{r} 458 \\ + 254 \\ \hline 712 \\ 10 \\ 100 \end{array}$$

Progressing to a short hand version of carrying.

$$\begin{array}{r} 247 \\ + 125 \\ \hline 372 \\ 1 \end{array} \quad \begin{array}{r} 458 \\ + 254 \\ \hline 712 \\ 1 \end{array}$$



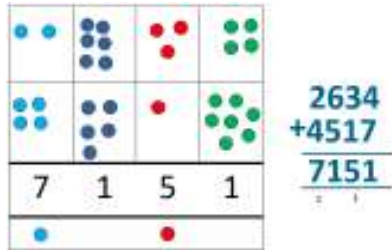
Written Methods of Addition



Objectives	Year 4	Guidance	Objectives	Year 5	Guidance	Objectives	Year 6 +	Guidance
	<p>Continue with a variety of missing number/digit problems with appropriate larger numbers.</p> <p><u>Mental and informal methods</u></p> <p>Children should be encouraged to decide if a mental strategy, supported by a range of models and images, including the number line and informal jottings, is more appropriate than a formal written method. The bar model should continue to be used to help with problem solving.</p> <p><u>Written methods (progressing to 4-digits)</u></p> <p>Expanded column addition modelled with place value counters, progressing to calculations with 4-digit numbers.</p> <div><div><div><div><div></div><div></div></div><div><div></div><div></div></div><div><div></div><div></div></div></div><div><div></div><div></div></div><div><div></div><div></div></div></div><div><div></div><div></div></div><div><div></div><div></div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div><div></div><div></div></div><div><div></div><div></div></div><div><div></div><div></div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div></div><div></div></div> <div><div><div></div><div></div></div><div><div></div><div></div></div><div><div></div><div></div></div></div> <div><div></div><div></div></div> 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Compact written method

Extend to numbers with at least four digits.


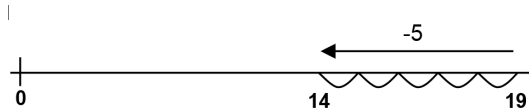
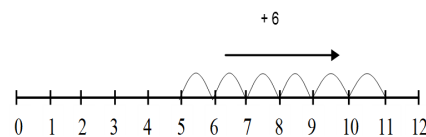
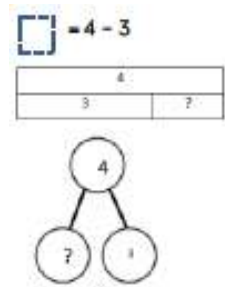
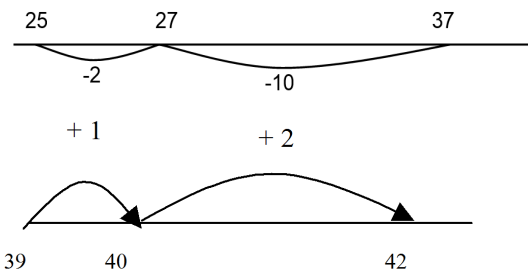
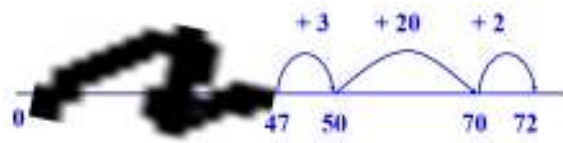
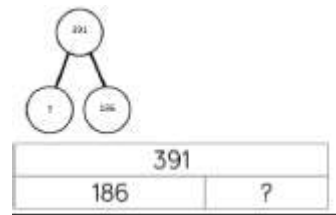
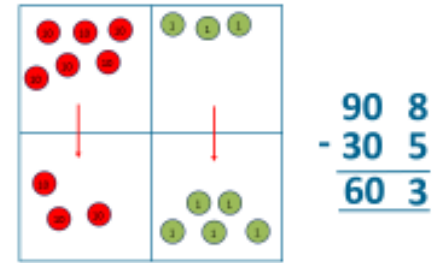


Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty.

Extend to up to two decimal places (same number of decimal digits in each number) in the contexts of measures or money and to adding several numbers (with different numbers of digits). Children should continue to carry below the line.

$$\begin{array}{r} \pounds 72.82 \\ + \pounds 54.63 \\ \hline \pounds 127.45 \\ 11 \end{array} \qquad \begin{array}{r} 782 \\ 26 \\ + 1237 \\ \hline \end{array}$$

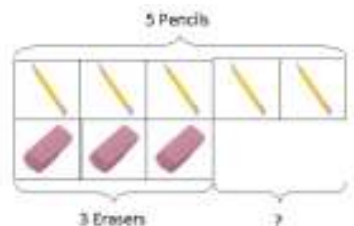
Written Methods of Subtraction

Objectives	Year 1	Guidance	Objectives	Year 2	Guidance	Objectives	Year 3	Guidance
	<p>Missing number problems e.g. $7 = \bigcirc - 9$; $20 - \bigcirc = 9$; $15 - 9 = \bigcirc$; $\bigcirc - \bigcirc = 11$.</p> <p>Use concrete objects and pictorial representations. If appropriate, progress using number lines with every number shown to number lines with significant numbers shown.</p> <p>Understand subtraction as take-away:</p>   <p>Understand subtraction as finding the difference:</p> 		<p>Missing number problems e.g. $52 - 8 = \bigcirc$; $\bigcirc - 20 = 25$; $22 = \bigcirc - 21$; $6 + \bigcirc + 3 = 11$</p>  <p>It is valuable to use a range of representations (also see Y1). Continue to use number lines to model take-away and difference.</p> <p>E.g.</p>  <p>The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference which is 25.</p> 	<p>Missing number problems e.g. $\bigcirc = 43 - 27$; $145 - \bigcirc = 138$; $274 - 30 = \bigcirc$; $245 - \bigcirc = 195$; $532 - 200 = \bigcirc$; $65 + \bigcirc + 126 = 254$.</p>  <p>Mental Strategies</p> <p>Children should start to develop, supported by a range of models and images, including the number line an understanding of when a mental or informal strategy is most appropriate. The bar model should be used to support problem solving (see Y1 and Y2). Children should make choices about whether to use complimentary addition or counting back, depending on the numbers involved.</p> <p>Written methods (progressing to 3-digits)</p> <p>Introduce expanded column subtraction with no decomposition (exchanging), modelled with place value counters (Dienes could be used for those who need a less abstract representation)</p> 				

$$6 - 2 = 4$$



Using the bar model to find the difference.



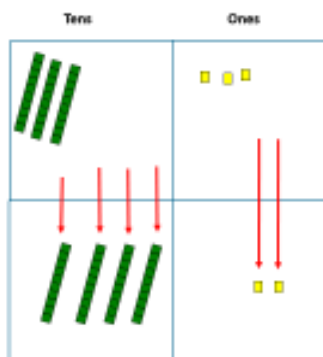
The above model would be introduced with concrete objects which children can move (including cards with pictures) before progressing to a pictorial representation. The use of other images is also valuable for modelling subtraction e.g. Numicon, bundles of straws, Dienes apparatus, multi-link cubes, bead strings.

The bar model should continue to be used, as well as images in the context of measure.



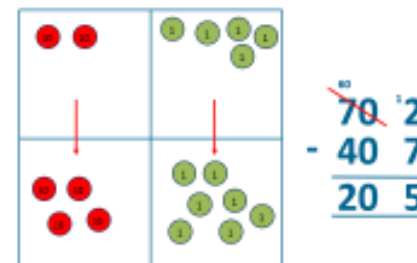
Towards written methods

Recording addition and subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus. E.g. $75 - 42$



$$\begin{array}{r} 70 \ 5 \\ - 40 \ 2 \\ \hline 30 \ 3 \end{array}$$

For some children this will lead to exchanging, modelled using concrete resources. E.g. place value counters (or Dienes)



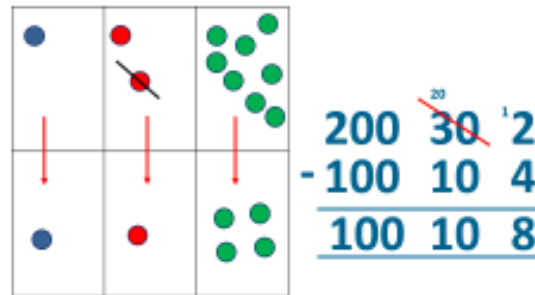
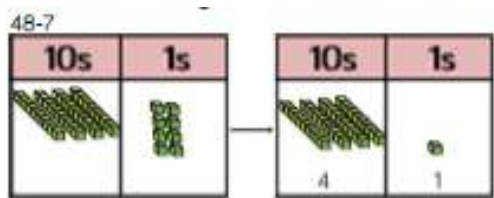
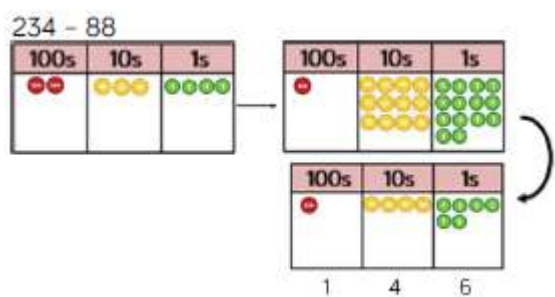
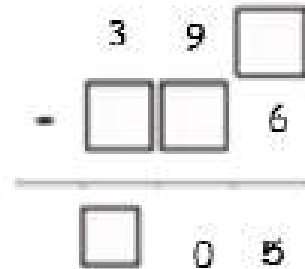
$$\begin{array}{r} 70 \ 2 \\ - 40 \ 7 \\ \hline 20 \ 5 \end{array}$$



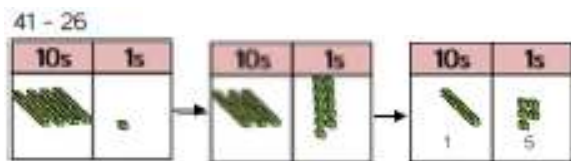
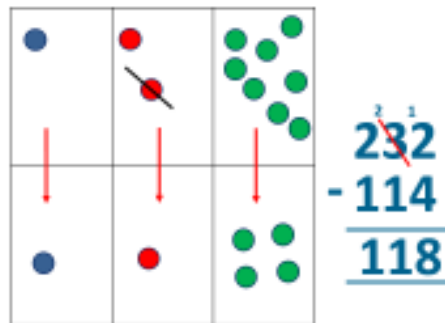
A number line and expanded column method may be compared next to each other.

Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

Written Methods of Subtraction

Objectives	Year 4	Guidance	Objectives	Year 5	Guidance	Objectives	Year 6 +	Guidance
	<p>Missing number/digit problems: $456 + \bigcirc = 710$; $1\bigcirc7 + 6\bigcirc = 200$; $60 + 99 + \bigcirc = 340$; $200 - 90 - 80 = \bigcirc$; $225 - \bigcirc = 150$; $\bigcirc - 25 = 67$; $3450 - 1000 = \bigcirc$; $\bigcirc - 2000 = 900$.</p> <p>Mental Strategies</p> <p>Pupils should continue to develop an understanding of when a mental or informal strategy is more efficient than a formal algorithm. They should support their work with a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.</p> <p>Written methods (progressing to 4-digits)</p> <p>Expanded column subtraction with decomposition, modelled with place value counters, progressing to calculations with 4-digit numbers.</p>  		<p>Missing number/digit problems: $6.45 = 6 + 0.4 + \bigcirc$; $119 - \bigcirc = 86$; $1000000 - \bigcirc = 999000$; $600000 + \bigcirc + 1000 = 671000$; $12462 - 2300 = \bigcirc$</p> <p>Mental Strategies</p> <p>Pupils should continue to develop an understanding of when a mental or informal strategy is more efficient than a formal algorithm. They should support their work with a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. This should be extended into work with decimals.</p> <p>Written methods (progressing to more the 4-digits)</p> <p>When understanding of the expanded method is secure, children will move on to the formal method of decomposition, which can be initially modelled with place value counters.</p> 		<p>Missing number/digit problems: \bigcirc and $\#$ each stand for a different number. $\# = 34$. $\# + \# = \bigcirc + \bigcirc + \#$. What is the value of \bigcirc? What if $\# = 28$? What if $\# = 21$? $10000000 = 9000100 + \bigcirc$ $7 - 2 \times 3 = \bigcirc$; $(7 - 2) \times 3 = \bigcirc$; $(\bigcirc - 2) \times 3 = 15$</p> 	<p>Mental Strategies</p> <p>Pupils should continue to develop an understanding of when a mental or informal strategy is more efficient than a formal algorithm. They should support their work with a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. This should be extended into work with decimals and simple fractions.</p> <p>Written methods</p> <p>As per year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with decomposition to be secured.</p>		

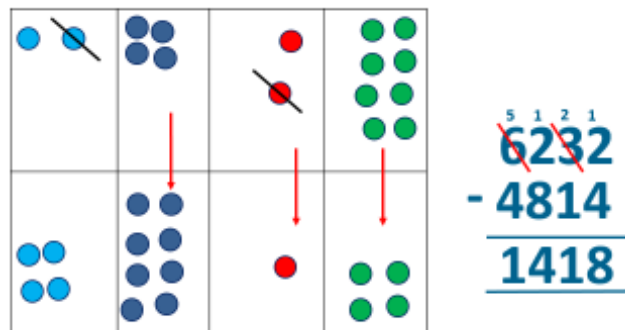
If understanding of the expanded method is secure, children will move on to the formal method of decomposition, which again can be initially modelled with the place value counters or Dienes.



Extend to up to two decimal places (same number of decimal digits in each number) in the contexts of measures or money.

$$\begin{array}{r} \text{£ } 27.58 \\ - \text{£ } 3.42 \\ \hline \text{£ } 24.16 \end{array}$$

or



Progress to calculating with decimals, including those with different numbers of decimal places.

$$\begin{array}{r} 234 \\ - 88 \\ \hline 6 \end{array}$$

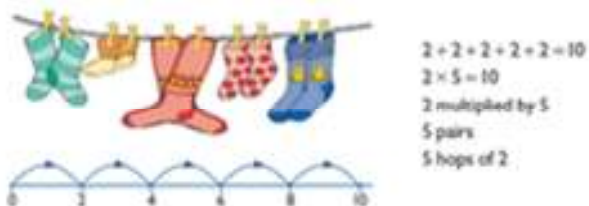
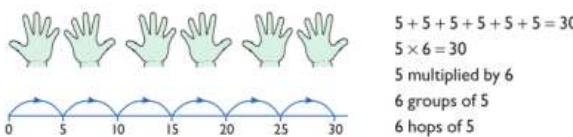
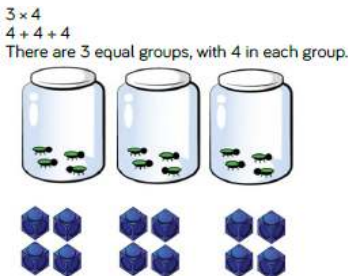
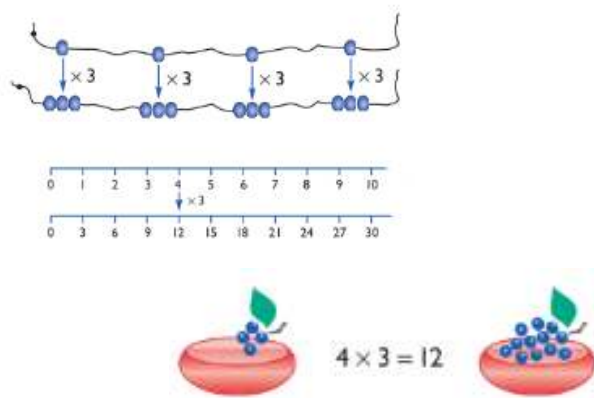
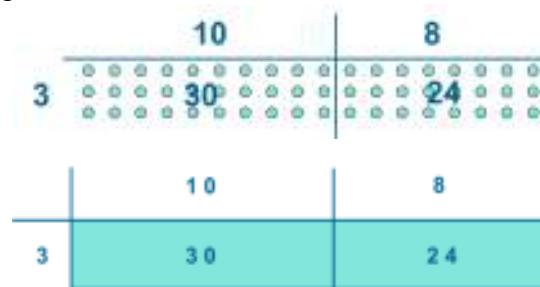
Children must understand what has happened when they have crossed out digits.

Continue calculating with decimals, including those with different numbers of decimal places. Extend to simple fractions with a common denominator.



Written Methods of Multiplication



Objectives	Year 1	Guidance	Objectives	Year 2	Guidance	Objectives	Year 3	Guidance
	<p>Understand multiplication is related to doubling and combining groups of the same size (repeated addition)</p> <p>Washing line, and other practical resources for counting 'lots of'. Concrete objects. Numicon; bundles of straws, bead strings.</p> <div></div> <div></div> <p>Problem solving with concrete objects (including money and measures)</p> <div></div>		<p>Expressing multiplication as a number sentence using x symbol.</p> <p>Using understanding of the inverse and practical resources to solve missing number problems.</p> <div>$7 \times 2 = \quad = 2 \times 7$$7 \times \quad = 14 \quad 14 = \quad \times 7$$\times 2 = 14 \quad 14 = 2 \times$$\times \bigcirc = 14 \quad 14 = \quad \times \bigcirc$</div> <p>Develop understanding of multiplication using arrays and number lines (see Year 1). Include multiplications not in the 2, 5 or 10 times tables.</p> <p>Begin to develop understanding of multiplication as scaling (3 times bigger/taller/heavier).</p> <div></div>		<p>Missing number problems. Continue with a range of missing number problems as in year two but with appropriate numbers.</p> <p>Mental strategies and informal methods</p> <p>Children should begin to understand when it is appropriate to use a mental or informal strategy:</p> <p>Doubling two digit numbers using partitioning.</p> <p>Demonstrating multiplication on a number line – jumping in larger groups of amounts.</p> <p>13 x 4 = 10 groups of 4 + 3 groups of 4.</p> <p>Written methods (progressing to 2-digit x 1-digit)</p> <p>Developing written methods using understanding of visual images.</p> <div></div> <p>Give children opportunities to explore this and deepen their understanding using Numicon, Dienes apparatus and place value counters.</p>			

Use cuisenaire and bar method to develop the vocabulary relating to 'times' –

Pick up five, 4 times. How many?

Visual representations

$$3 \times 4$$



Use arrays to understand multiplication can be done in any order (commutative)

$$2 \times 5 = 5 \times 2$$



2 lots of 5



5 lots of 2

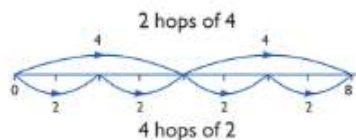


$$2 \times 4 = 8$$



$$4 \times 2 = 8$$

$$2 \times 4 = 8$$



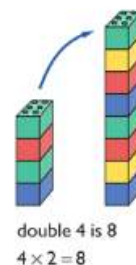
Doubling numbers up to 10.

Link with understanding scaling.

Using known doubles to double

2-digit numbers.

(double 15 = double 10 + double 5)



Towards written methods

Use jottings to develop an understanding of doubling two digit numbers.

$$\begin{array}{cc} & 16 \\ & \swarrow \searrow \\ 10 & 6 \\ | & | \\ \times 2 & \times 2 \\ \hline 20 & + & 12 = 32 \end{array}$$

$$\begin{array}{r} 4 \times 15 \\ \swarrow \searrow \\ 10 \quad 5 \end{array}$$

$$10 \times 4 = 40$$

$$5 \times 4 = 20$$

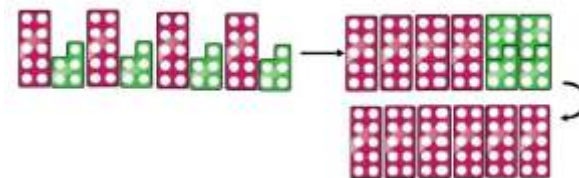
$$40 + 20 = 60$$

A number line can also be used



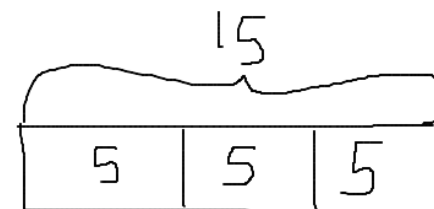
Partition to multiply

$$4 \times 15$$



Children could use bar modelling as an informal method of recording.

$$3 \times 5 =$$



Place value counters can be used to introduce a more formal method. E.g. 23×3

10s	1s
6	9

10s	1s
00	000
00	000
00	000
6	9

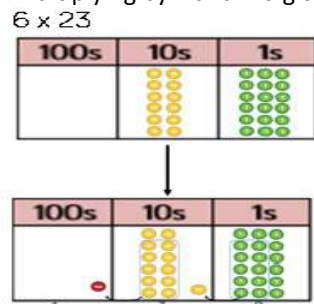
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Written Methods of Multiplication



Objectives	Year 4	Guidance	Objectives	Year 5	Guidance	Objectives	Year 6 +	Guidance
	<p>Continue with a range of equations as in Year 2 and 3 but with appropriate numbers. Also include calculations with missing digits.</p> <p>○2 x 5 = 160</p> <p>Mental Strategies</p> <p>Children continue to identify when it is better to use a mental or informal strategy to solve questions.</p> <p>E.g. 45 x 9 = 45 x 10 – 45</p> <p>Counting in multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.</p> <p>Solving practical problems where children need to scale up. Relate to known number facts. E.g. How tall would a 25cm sunflower be if it grew 6 times taller?</p> <p>Written methods (progressing to 3-digit x 2-digit)</p> <p>Using expanded notation to multiply 2-digit and 3-digit numbers by a single digit.</p> <div><div><div>35</div><div>X 7</div><div>5 x 7 35</div><div>30 x 7 210</div><div>245</div></div><div><div>264</div><div>x 5</div><div>4 x 5 20</div><div>60x5 300</div><div>200x5 1000</div><div>1320</div></div></div>		<p>Continue with a range of equations as in years 2 to 4 but with appropriately sized numbers (4 digits by up to 2 digits). Include calculations with missing digits.</p> <div><div>246</div><div>X ○</div><div>1004</div></div> <div>2○ x 3 = 63</div>	<p>Mental strategies</p> <p>Children continue to identify when it is better to use a mental or informal strategy to solve questions.</p> <p>Multiply by 10, 100, 1000 extend to multiply by 0.1.</p> <p>Use practical resources and jottings to explore equivalent statements. E.g. 4 x 35 = 2 x 2 x 35</p> <p>Recall prime numbers to 19 and identify prime numbers to 100 (with reasoning)</p> <p>Solving practical problems where children need to scale up. Relate to known number facts.</p> <p>Identify factor pairs for numbers by relating to known multiplication facts.</p>		<p>Continue with a range of equations as in years 2 to 4 but with appropriately sized numbers (4 digits by 2 digits). Include calculations with missing digits of increasing complexity.</p> <div><div>236</div><div>x 2○</div><div>9○4</div><div>4○20</div><div>○664</div></div> <p>Mental Strategies</p> <p>Children continue to identify when it is better to use a mental or informal strategy to solve questions.</p> <p>Identifying common factors and multiples of given numbers.</p> <p>Solving practical problems where children need to scale up. Relate to known number facts.</p> <p>Written Methods</p> <p>Continue to refine and deepen understanding of written methods with emphasis on developing fluency in using long multiplication.</p> <div><div><div>124</div><div>x 26</div><div>- 7 4 4</div><div>2 4 8 0</div><div>3 2 2 4</div><div>1 1</div></div><div>Answer: 3224</div></div>		

Using place value counters to reinforce the concept of carrying when multiplying by 1 and 2-digit values.



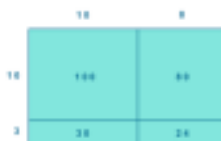
Children start to record their work using an expanded notation.

$$\begin{array}{r}
 6 \times 23 \\
 \quad 23 \\
 \times \quad 6 \\
 \hline
 3 \times 6 \quad 18 \\
 20 \times 6 \quad 120 \\
 \hline
 138
 \end{array}$$

A formal method should be introduced alongside expanded notation as a way to streamline calculations not as a new method.

$$\begin{array}{r}
 6 \times 23 = \\
 \quad 23 \\
 \times \quad 6 \\
 \hline
 138 \\
 \text{1 1}
 \end{array}$$

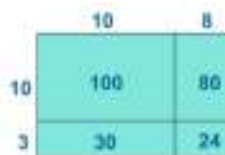
Some children may explore use of grid method to investigate multiplying 2-digit by 2-digit numbers using known tables facts.



Written methods (progressing to 4-digit x 2-digit)

Children extend their use of expanded notation to multiply a 2-digit number by a 2-digit number. This may be built on their exploration of multiplying two 2-digit numbers using known tables facts and grid method.

$$\begin{array}{r}
 18 \\
 \times 13 \\
 \hline
 3 \times 8 \quad 24 \\
 3 \times 10 \quad 30 \\
 10 \times 8 \quad 80 \\
 10 \times 10 \quad 100 \\
 \hline
 234 \\
 \text{1}
 \end{array}$$



A formal method should be introduced alongside expanded notation as a way to streamline calculations not as a new method. This may cross over into the beginning of Year 6.

$$\begin{array}{r}
 18 \\
 \times 13 \\
 \hline
 54 \\
 \text{2} \\
 180 \\
 234
 \end{array}$$

$$\begin{array}{r}
 236 \\
 \times 24 \\
 \hline
 944 \\
 \text{1 2} \\
 4720 \\
 \hline
 5664 \\
 \text{1}
 \end{array}$$

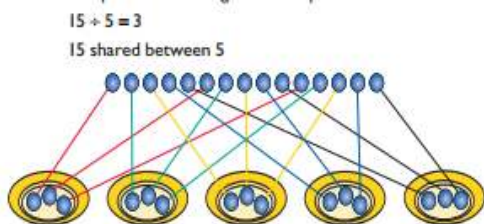
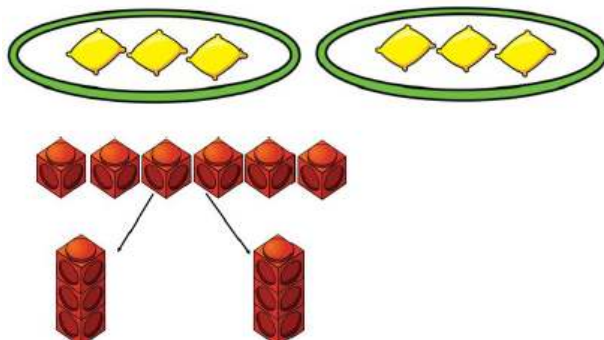
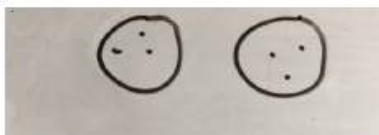
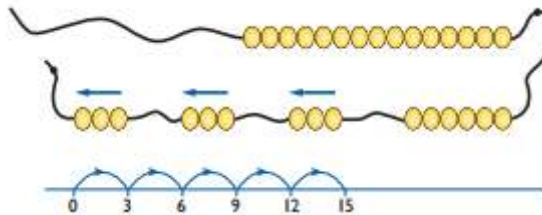
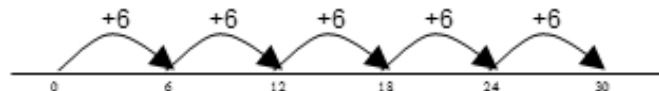
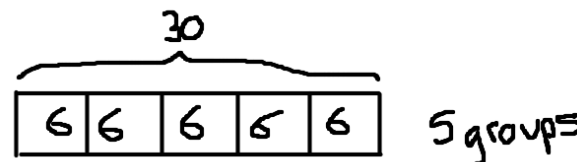
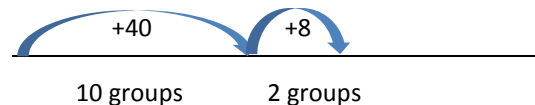
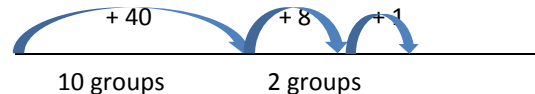
Extend to multiplying a 1-digit number with up to two decimal places by a 2-digit or 3-digit whole number.

Some children may extend to multiplying a 2-digit or 3-digit integer by a single decimal value. E.g. 247 x 2.3

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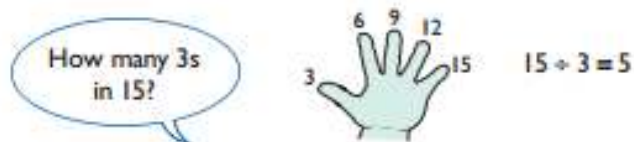

Written Methods of Division



Objectives	Year 1	Guidance	Objectives	Year 2	Guidance	Objectives	Year 3	Guidance
	<p>Children must have secure counting skills – being able to confidently count in 2s, 5s and 10s.</p> <p>Children should be given opportunities to reason about what they notice in number patterns.</p> <p><u>Group AND share small quantities – understanding the difference between the two concepts.</u></p> <p><u>Sharing</u></p> <p>Develops importance of one-to-one correspondence.</p>  <p>Children should be taught to share using concrete apparatus.</p> <p>$6 \div 2$</p> 		<p><u>÷ = signs and missing numbers</u></p> <p>$6 \div 2 = \bigcirc$; $\bigcirc = 6 \div 2$; $6 \div \bigcirc = 3$; $3 = 6 \div \bigcirc$</p> <p>Know and understand the concepts of sharing and grouping (see year 1) and pupils should know division is shown with the ÷ sign.</p> <p>Children should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations.</p> <p>$6 \div 2$</p>  <p><u>Grouping using a number-line</u></p> <p>Group from zero in jumps of the divisor to find our ‘how many groups of three are there in 15?’</p> <p>$15 \div 3 = 5$</p> 		<p><u>÷ = signs and missing numbers</u></p> <p>Continue using a range of equations as in year 2 but with appropriate numbers.</p> <p><u>Grouping</u></p> <p>How many 6’s are in 30? $30 \div 6$ can be modelled as:</p>   <p><u>Becoming more efficient using a number line</u></p> <p>Children need to be able to partition the dividend in different ways.</p> <p>$48 \div 4 = 12$</p>  <p><u>Remainders</u></p> <p>$49 \div 4 = 12r1$</p> 			

Grouping

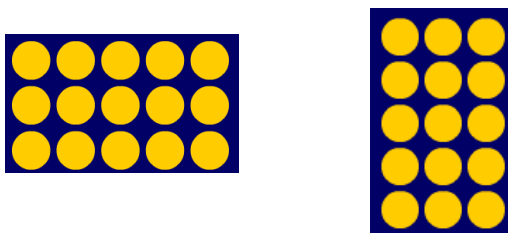
Children should apply their counting skills to develop some understanding of grouping.



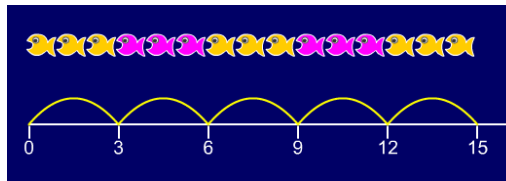
Use of arrays as a pictorial representation for division.

$15 \div 3 = 5$. There are 5 groups of 3.

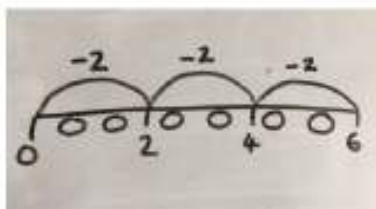
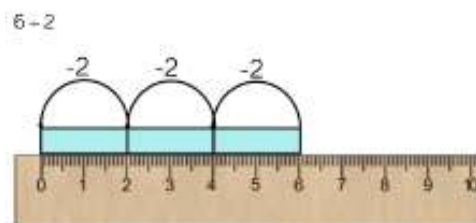
$15 \div 5 = 3$. There are 3 groups of 5.



Children should be able to find simple fractions, $\frac{1}{2}$ and $\frac{1}{4}$ of a set of objects, numbers and quantities.



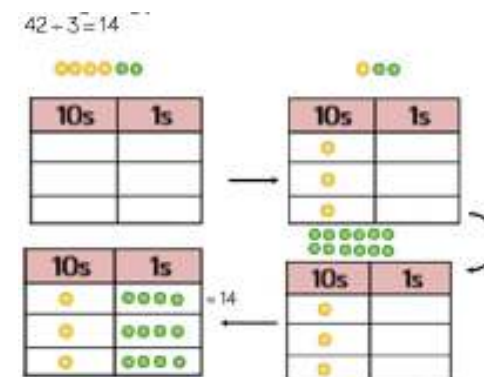
Repeated subtraction – Cuisenaire rods over a number line.



Continue to work on arrays. Support children to understand how multiplication and division are inverse. Look at the array – what do you see?

Sharing – 49 shared between 4. How many are left over?
Grouping – how many 4's are in 49? How many left over?

Sharing using place value counters.



Place value counters can be used to support children apply their knowledge of grouping.

For example:

$130 \div 10 =$ how many groups of 10 in 130?

$1400 \div 100 =$ how many groups of 100 in 1400?

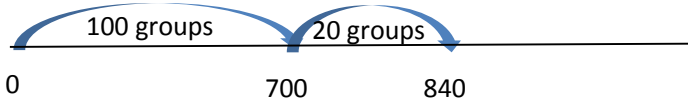
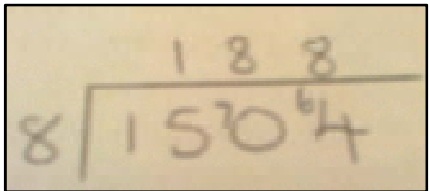
Some children may start to use short division when dividing 2-digit numbers by a single digit without a remainder.

$$\begin{array}{r} 13 \\ 6 \overline{) 78} \end{array}$$

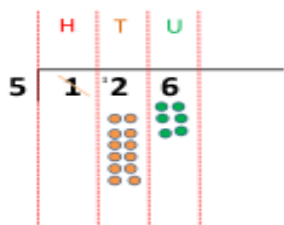


Written Methods of Division



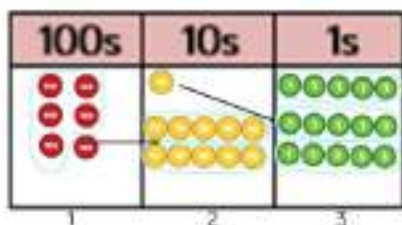
Objectives	Year 4	Guidance	Objectives	Year 5	Guidance	Objectives	Year 6 +	Guidance
	<p><u>÷ = signs and missing numbers</u></p> <p>Continue using a range of equations as in year 3 but with appropriate numbers.</p> <p>Continue to solve missing number/digit problems for pupils to solve including problems in context. E.g. Tim shares sweets with his friends. Each gets 6 sweets. If Tim started with 30 sweets how many people got sweets?</p> <p><u>Sharing, grouping and using a number line</u></p> <p>Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. Children should progress in their use of written division calculations:</p> <ul style="list-style-type: none">Using tables facts with which they are fluent.Experiencing a logical progression in the numbers they use, for example:<ol style="list-style-type: none">Dividend just over 10x the divisor, e.g. $84 \div 7$Dividend just over 10x the divisor when the divisor is a teen number, e.g. $173 \div 15$ (learning sensible strategies for calculations such as $102 \div 17$)Dividend over 100x the divisor. E.g. $840 \div 7$Dividend over 20x the divisor, e.g. $168 \div 7$ <p>All of the above stages should include calculations with remainders as well as without.</p> <p>Remainders should be interpreted according to the context. (i.e. rounded up or down to relate to the answer to the problem)</p> <div><div><p>e.g. $840 \div 7 = 120$</p><p><u>Jottings</u></p><p>$7 \times 100 = 700$ $7 \times 10 = 70$ $7 \times 20 = 140$</p></div></div>					<p><u>÷ = signs and missing numbers</u></p> <p>Continue using a range of equations as previously but with appropriate numbers. Continue to solve missing number/digit problems including those in context as in years 4 and 5.</p> <p><u>Sharing, grouping and using a number line</u></p> <p>Children will continue to explore division as sharing and grouping, and to represent calculations on a number line as appropriate.</p> <p>Quotients with remainders should be expressed as fractions or decimals as appropriate to the question.</p> <p><u>Mental Strategies</u></p> <p>Children should consider when it is more appropriate to use a mental or informal strategy. E.g. when dividing by 2, 4, 8, 5, 10, 100 etc.</p> <p><u>Formal written methods – long and short division</u></p> <p>Children should identify when it is appropriate to use short division (when the divisor is a single digit) and when to use long division (when the divisor is greater than 10 but not a multiple of ten)</p> <p>Children should know that when the divisor is a multiple of 10 to partition and then divide. E.g. $364 \div 20 = 364 \div 10 \div 2$</p> <p>Short division. E.g. $1504 \div 8$</p> 		
	<p><u>Formal written methods</u></p> <p>Formal short division should only be introduced once the children have a good understanding of division, its links with multiplication and the idea of ‘chunking up’ (grouping) to find a target number. (see use of number lines above)</p>		<p><u>Formal written methods</u></p> <p>Continued as shown in year 4, leading to the efficient use of a formal method of short division. The language of grouping should be used.</p> <p>Children recognise that the method of short division is only used when the divisor is a single digit and mental strategies for dividing (e.g. by 2 and 4) should always be considered first.</p>					

Short division to be modelled for understanding using place value counters. Calculations with 2 and 3-digit dividends by a single digit divisor.

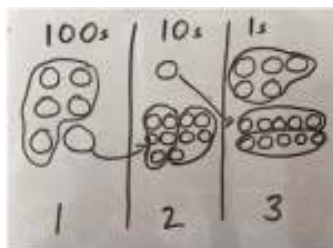


Or

$$615 \div 5$$



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?



Short division

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$

$$1435 \div 6$$

$$\begin{array}{r} 239 \text{ r } 1 \\ 6 \overline{) 1435} \end{array}$$

Children begin to practically develop their understanding of how to express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. What could I do with this remaining 1? How could I share this between 6 as well?

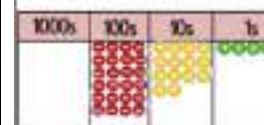
Long division using place value counters
 $2544 \div 12$



We can't group 2 thousands into groups of 12 so we'll exchange them.

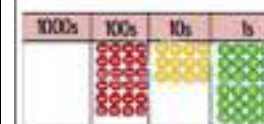
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 211 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 groups of 12, which leaves no remainder.

$$\begin{array}{r} 211.2 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Long division. E.g. $2364 \div 15$

$$\begin{array}{r} 157.6 \\ 15 \overline{) 2364.0} \\ \underline{15} \\ 86 \\ \underline{75} \\ 114 \\ \underline{105} \\ 90 \\ \underline{90} \\ 0 \end{array}$$

$$\begin{aligned} 2364 \div 15 &= 157 \text{ r } 9 \\ &= 157 \frac{9}{15} \text{ (expressing remainder as a fraction)} \\ &= 157.6 \text{ (expressing remainder as a decimal)} \end{aligned}$$

Problem Solving with Bar Models

Key steps in bar modelling:

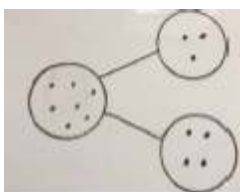
1. Read the problem and write an answer statement.
2. Identify who and what is involved in the problem.
3. Draw unit bar(s) of equal length.
4. Chunk up the bars with the information from the problem:
 - Do you know the value of the whole?
 - How many parts?
 - Do you know the value of any parts?
5. Solve the problem.
6. Re-read the question and check calculations.
7. Answer the question by completing the answer statement.

Key Images in Bar Modelling

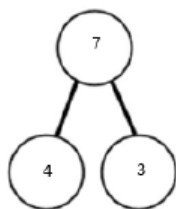
Younger children should become familiar with part-whole models before using a more formal bar model.

Part-whole models may be introduced from reception onwards.

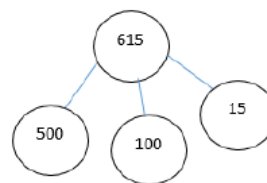
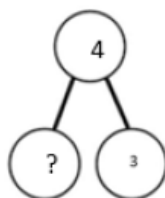
E.g. Pictorially



Abstract



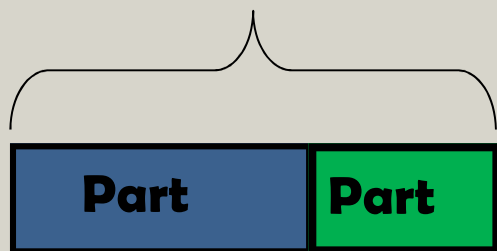
and in different forms and orientations.



There are five key structures of bar model that children should be familiar with:

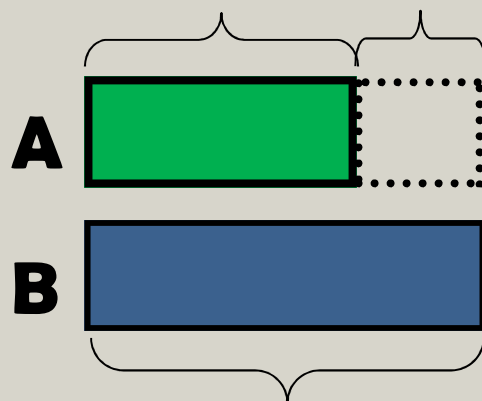
Part-Part-Whole

Whole



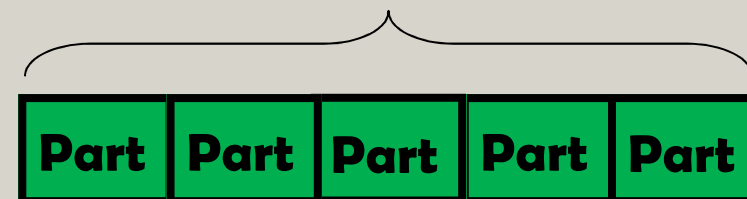
Comparison

DIFFERENCE



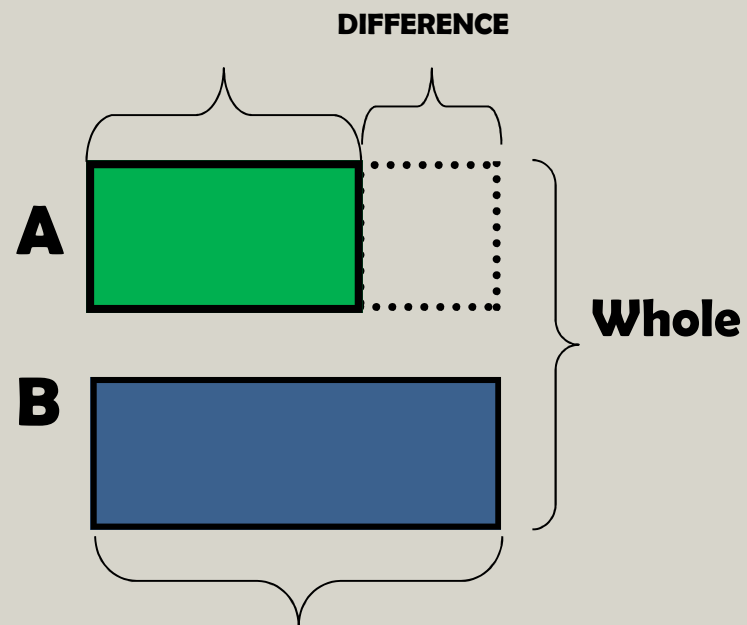
Equal Parts of a Whole

Whole

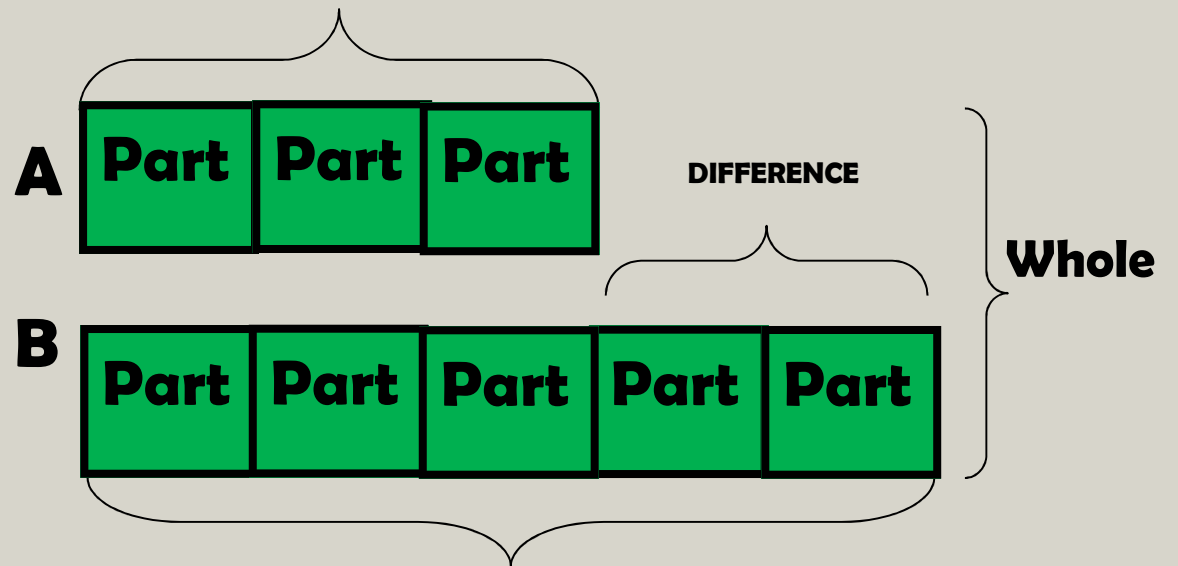


An Equal part is a UNIT

Comparison AND Part-Part-Whole



Comparison AND Equal Parts of Wholes



An Equal part is a UNIT

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National Curriculum 2014 Objectives and Guidance

Year 1 objectives, + & -

Statutory requirements

Pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$.

[Back \(+\)](#)

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Year 1 Guidance, + & -

Notes and guidance (non-statutory)

Pupils memorise and reason with number bonds to 10 and 20 in several forms (for example, $9 + 7 = 16$; $16 - 7 = 9$; $7 = 16 - 9$). They should realise the effect of adding or subtracting zero. This establishes addition and subtraction as related operations.

Pupils combine and increase numbers, counting forwards and backwards.

They discuss and solve problems in familiar practical contexts, including using quantities. Problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

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Year 2 Objectives, + & -

Statutory requirements

Pupils should be taught to:

- solve problems with addition and subtraction:
 - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
 - applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and ones
 - a two-digit number and tens
 - two two-digit numbers
 - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

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Year 2 Guidance, + & -

Notes and guidance (non-statutory)

Pupils extend their understanding of the language of addition and subtraction to include sum and difference.

Pupils practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as using $3 + 7 = 10$; $10 - 7 = 3$ and $7 = 10 - 3$ to calculate $30 + 70 = 100$; $100 - 70 = 30$ and $70 = 100 - 30$. They check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, $5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5$). This establishes commutativity and associativity of addition.

Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.

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[See also Interim Teacher Assessments KS1](#)

Year 3 Objectives, + & -

Statutory requirements

Pupils should be taught to:

- add and subtract numbers mentally, including:
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

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Year 3 Guidance, + & -

Notes and guidance (non-statutory)

Pupils practise solving varied addition and subtraction questions. For mental calculations with two-digit numbers, the answers could exceed 100.

Pupils use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent (see [Mathematics Appendix 1](#)).

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Year 4 Objectives, + & -

Statutory requirements

Pupils should be taught to:

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

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Year 4 Guidance, + & -

Notes and guidance (non-statutory)

Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency (see [English Appendix 1](#)).

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Year 5 Objectives, + & -

Statutory requirements

Pupils should be taught to:

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

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Year 5 Guidance, + & -

Notes and guidance (non-statutory)

Pupils practise using the formal written methods of columnar addition and subtraction with increasingly large numbers to aid fluency (see [Mathematics Appendix 1](#)).

They practise mental calculations with increasingly large numbers to aid fluency (for example, $12\ 462 - 2300 = 10\ 162$).

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Year 6 Objectives, + & -

Statutory requirements

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

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Mathematics

Statutory requirements

- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

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Year 6 Guidance, + & -

Notes and guidance (non-statutory)

Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division (see [Mathematics Appendix 1](#)).

They undertake mental calculations with increasingly large numbers and more complex calculations.

Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.

Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.

Pupils explore the order of operations using brackets; for example, $2 + 1 \times 3 = 5$ and $(2 + 1) \times 3 = 9$.

Common factors can be related to finding equivalent fractions.

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[See also Interim Teacher Assessments KS2](#)

Year 1 Objectives, \times & \div

Statutory requirements

Pupils should be taught to:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

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Year 1 Guidance, \times & \div

Notes and guidance (non-statutory)

Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities.

They make connections between arrays, number patterns, and counting in twos, fives and tens.

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[Back \(\$\div\$ \)](#)

Year 2 Objectives, \times & \div

Statutory requirements

Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals ($=$) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

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[Back \(\$\div\$ \)](#)

Year 2 Guidance, \times & \div

Notes and guidance (non-statutory)

Pupils use a variety of language to describe multiplication and division.

Pupils are introduced to the multiplication tables. They practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.

Pupils work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. They begin to relate these to fractions and measures (for example, $40 \div 2 = 20$, 20 is a half of 40). They use commutativity and inverse relations to develop multiplicative reasoning (for example, $4 \times 5 = 20$ and $20 \div 5 = 4$).

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[See also Interim Teacher Assessments KS1](#)

Year 3 Objectives, \times & \div

Statutory requirements

Pupils should be taught to:

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

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Year 3 Guidance, \times & \div

Notes and guidance (non-statutory)

Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.

Pupils develop efficient mental methods, for example, using commutativity and associativity (for example, $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and multiplication and division facts (for example, using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts (for example, $30 \times 2 = 60$, $60 \div 3 = 20$ and $20 = 60 \div 3$).

Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.

Pupils solve simple problems in contexts, deciding which of the four operations to use and why. These include measuring and scaling contexts, (for example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

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Year 4 Objectives, x & ÷

Statutory requirements

Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to 12×12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

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Year 4 Guidance, \times & \div

Notes and guidance (non-statutory)

Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency.

Pupils practise mental methods and extend this to three-digit numbers to derive facts, (for example $600 \div 3 = 200$ can be derived from $2 \times 3 = 6$).

Notes and guidance (non-statutory)

Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers (see [Mathematics Appendix 1](#)).

Pupils write statements about the equality of expressions (for example, use the distributive law $39 \times 7 = 30 \times 7 + 9 \times 7$ and associative law $(2 \times 3) \times 4 = 2 \times (3 \times 4)$).

They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, $2 \times 6 \times 5 = 10 \times 6 = 60$.

Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or three cakes shared equally between 10 children.

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Year 5 Objectives, x & ÷

Statutory requirements

- recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Statutory requirements

Pupils should be taught to:

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

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Year 5 Guidance, x & ÷

Notes and guidance (non-statutory)

Pupils practise and extend their use of the formal written methods of short multiplication and short division (see [Mathematics Appendix 1](#)). They apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.

They use and understand the terms factor, multiple and prime, square and cube numbers.

Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, $98 \div 4 = \frac{98}{4} = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5 \approx 25$).

Pupils use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres.

Distributivity can be expressed as $a(b + c) = ab + ac$.

They understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, $4 \times 35 = 2 \times 2 \times 35$; $3 \times 270 = 3 \times 3 \times 9 \times 10 = 9^2 \times 10$).

Pupils use and explain the equals sign to indicate equivalence, including in missing number problems (for example, $13 + 24 = 12 + 25$; $33 = 5 \times \square$).

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Year 6 Objectives, x & ÷

Statutory requirements

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Statutory requirements

- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

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Year 6 Guidance, x & ÷

Notes and guidance (non-statutory)

Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division (see [Mathematics Appendix 1](#)).

They undertake mental calculations with increasingly large numbers and more complex calculations.

Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.

Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.

Pupils explore the order of operations using brackets; for example, $2 + 1 \times 3 = 5$ and $(2 + 1) \times 3 = 9$.

Common factors can be related to finding equivalent fractions.

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[See also Interim Teacher Assessments KS2](#)